Problem Set 3
Computational and Metric Geometry

You can discuss homework problems with other students taking the class but you must write solutions on your own. This homework is due on Wednesday, March 13.

Problem 1. Prove that for every $\varepsilon > 0$ there exists C_ε such that every finite metric space X embeds into $\ell_{C_\varepsilon \log(|X|+1)}$ with distortion at most $1 + \varepsilon$.

Problem 2. Prove that for every k there exists a polynomial-time approximation algorithm $\mathcal{A}_k$ for Minimum Multicut with k terminal pairs that gives a $3/2$ approximation.

Problem 3. Suppose that we need to deliver packages from a warehouse s to a set of clients X. We have one truck that can carry at most W pounds at a time. At every trip, it leaves the warehouse with several packages of weight at most W, delivers them to the clients, and then gets back to the warehouse. The truck can make several trips. We are given the weights of all packages (the weight of the package for a client x is w_x) and the set of all distances. The distance from the warehouse s to a client x is $d(s, x)$; the distance between clients x and y is $d(x, y)$ (where d is a metric on $X \cup \{s\}$). The goal is to find the shortest route for the truck.

Give an $O(\log n)$ approximation algorithm for the problem.

- Design a constant factor approximation algorithm for a tree metric d.

  Hint: How many times does the optimal route goes along an edge (v, w)? Give a lower bound depending on the total weight of the packages that should be delivered to the clients located in the subtree rooted at w.

- Give an $O(\log n)$ approximation algorithm for a general metric space.

  Hint: Embed the metric space into a distribution of dominating trees (we will talk about embeddings into dominating trees on March 4).