Problem Set 1
Computational and Metric Geometry

You can discuss homework problems with other students, but you must write solutions on your own. This homework is due on Monday, February 11.

Definition. The ℓ_∞ *distance* between two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ is equal to $\|u - v\| = \max(|u_1 - v_1|, |u_2 - v_2|)$. The ℓ_∞ *distance* between two non-empty sets of points $U \subset \mathbb{R}^2$ and $V \subset \mathbb{R}^2$ is

$$d(U, V) = \inf_{u \in U, v \in V} \|u - v\|_\infty = \inf_{u \in U, v \in V} \max(|u_1 - v_1|, |u_2 - v_2|).$$

Problem 1. We are given a set of n points in the plane. Design an algorithm that finds a pair of points with maximum ℓ_∞ distance in time $O(n)$. Prove the correctness of your algorithm.

Problem 2. Design an algorithm that given a set of n axis-parallel rectangles in the plane finds a pair of rectangles with minimum ℓ_∞ distance in time $O(n \log n)$. Prove the correctness of your algorithm.
Partial credit: Solve the following simpler problem for a partial credit. Design an algorithm that given a set of n axis–parallel rectangles in the plane and a parameter t finds a pair of rectangles with ℓ_∞ distance less than t in $O(n \log n)$ time. If there is no such pair of rectangles, the algorithm should output that. Prove the correctness of your algorithm.

Problem 3. Design an algorithm that solves the following problem in $O(n \log n)$ time. We are given a rectangle $[0, A] \times [0, B]$ and a set of n unit disks with centers $(x_1, y_1), \ldots, (x_n, y_n)$. Disks may overlap. Determine if the disks cover the entire rectangle.

Prove the correctness of your algorithm.

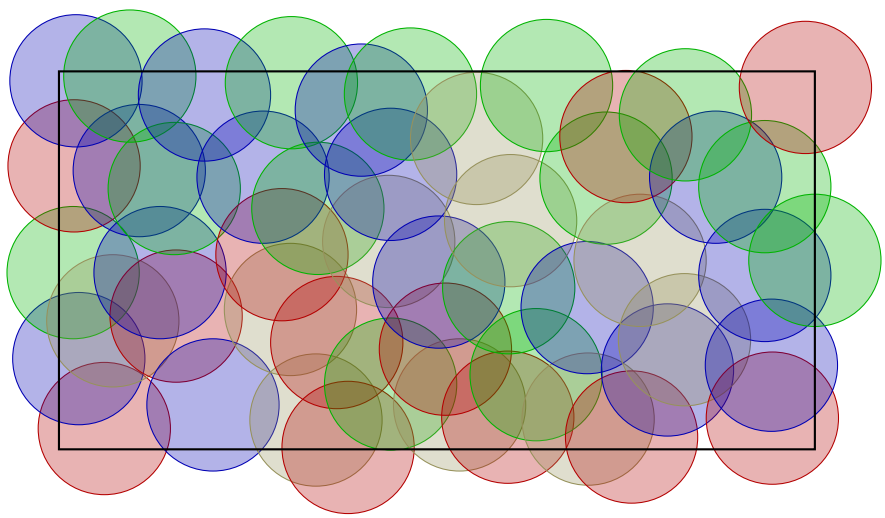

Figure 1: In this example, unit disks don't cover the entire rectangle.