Homework Assignment 2

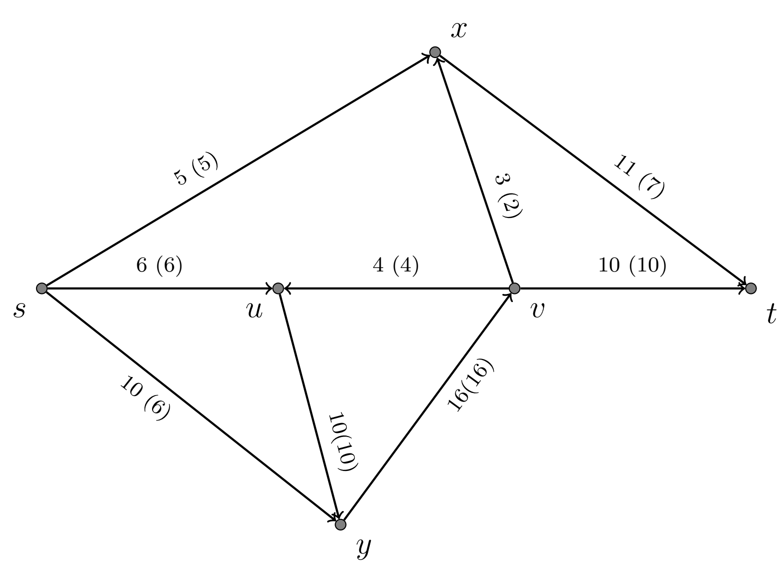
TTIC 31010

January 28, 2013

Problem 1. Let $G = (V, E)$ be an arbitrary directed graph, with a source s, a sink t, and a positive integer capacity $c(e)$ on every edge $e \in E$. Decide whether each of the following statements is true or false. If a statement is true, give a proof, and if it is false, show a counterexample.

- If all capacities $c(e)$ are even, then the value of the maximum flow is even.
- If all capacities $c(e)$ are odd, then the value of the maximum flow is odd.
- If f is a maximum s-t flow in G, then f saturates every edge in $out(s)$ with flow. That is, for each $e \in out(s)$, $f(e) = c(e)$.

Problem 2. Consider the flow network G shown in the figure below. For every edge e, its capacity $c(e)$ and its flow value $f(e)$ are written next to the edge ($f(e)$ appears in parentheses).



Part a. Is the flow f a maximum flow in the graph? Prove your answer.
Part b. What is the maximum flow value? Prove your answer.

Problem 3. Let $G = (V, E)$ be a directed graph, with source $s \in V$, sink $t \in V$, and non-negative edge capacities $\{c(e)\}$. Let $f : E \to \mathbb{R}_{\geq 0}$ be a maximum flow in G. Let G_f be the residual graph. Denote by S the set of nodes reachable from s in G_f and by T the set of nodes from which t is reachable in G_f. That is,

$$S = \{u : \text{ there is a directed path from } s \text{ to } u \text{ in } G_f\},$$
$$T = \{v : \text{ there is a directed path from } v \text{ to } t \text{ in } G_f\}.$$

Prove that $V = S \cup T$ if an only if G has a *unique* s-t minimum cut (an s-t cut whose capacity is strictly less than the capacity on any other s-t cut).