

Homework Assignment 1

TTIC 31010

January 14, 2014

Problem 1. We are given a set of n points $X = \{x_1, \dots, x_n\}$ on the real line. Give an algorithm that finds a minimum cardinality set of unit intervals that cover all points in X . Prove that the algorithm is correct. Find its running time.

Problem 2. Suppose we have an alphabet with 2^k characters, and a string S in which all characters are almost equally common. That is,

$$\min_{x \in \Sigma} p(x) > \frac{1}{2} \max_{x \in \Sigma} p(x)$$

where $p(x)$ is the frequency with which x appears in S . How does the Huffman tree for Σ look like? What is its cost? Prove your answer.

Problem 3. Given an unlimited supply of coins of denominations $x_1, x_2, \dots, x_n$ (where $x_1, \dots, x_n$ are positive integer numbers), we wish to make change for a value v ; that is, we wish to find a set of coins whose total value is v (the set may contain several coins of the same denomination). This might not be possible: for instance, if the denominations are 5 and 10 then we can make change for 15 but not for 12. Give an $O(nv)$ dynamic programming algorithm that does the following.

1. The algorithm determines if there is a set of coins of total value v .
2. If there is such set, the algorithm finds a set with the minimal possible number of coins.

Prove that the algorithm is correct.

Problem 4. Suppose we are given three strings A , B , and C over a finite alphabet Σ . Denote the i -th character of A by a_i . Let $n = |A|$ be the length of A .

$$A = a_1 a_2 \dots a_n$$

We say that string A is an *interleaving* of strings B and C if it is possible to partition the set of indices $\{1, \dots, n\}$ into two disjoint subsets $I = \{i_1, \dots, i_k\}$ and $J = \{j_1, \dots, j_{n-k}\}$ where $i_1 < \dots < i_k$ and $j_1 < \dots < j_{n-k}$ such that

$$B = a_{i_1} \dots a_{i_k}$$

$$C = a_{j_1} \dots a_{j_{n-k}}.$$

Find an efficient algorithm that given strings A , B and C decides whether A is an interleaving of B and C . Prove that the algorithm is correct. Find its running time.