Lecture 4
Backpropagation
CMSC 35246: Deep Learning

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&
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University of Chicago

April 5, 2017

- Things we will look at today
    - More Backpropagation

- Things we will look at today
  - More Backpropagation
  - Still more backpropagation

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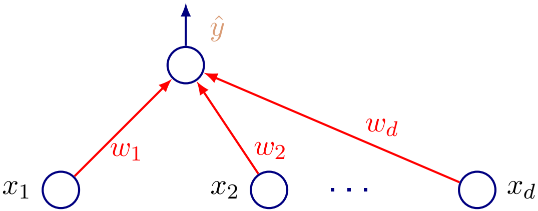
  - More Backpropagation
  - Still more backpropagation
  - Quiz at 4:05 PM

To understand, let us just calculate!

One Neuron Again



- Consider example $\mathbf{x}$; Output for $\mathbf{x}$ is $\hat{y}$; Correct Answer is y
- Loss $L = (y - \hat{y})^2$

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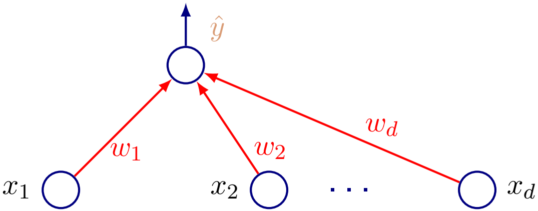
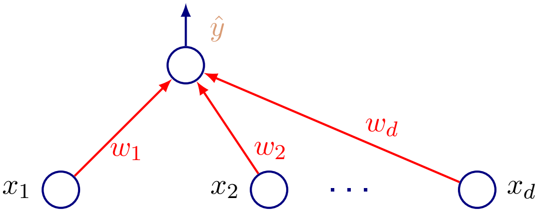


- Consider example $\mathbf{x}$; Output for $\mathbf{x}$ is $\hat{y}$; Correct Answer is y
- Loss $L = (y - \hat{y})^2$
- $\hat{y} = \mathbf{x}^T \mathbf{w} = x_1 w_1 + x_2 w_2 + \dots x_d w_d$

One Neuron Again



- Want to update w_i (forget closed form solution for a bit!)

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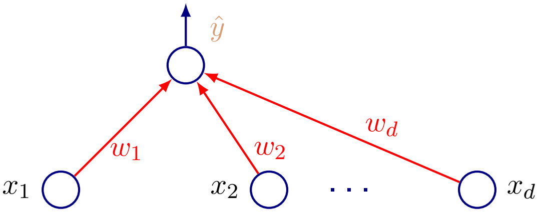

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- Update rule: $w_i := w_i - \eta \frac{\partial L}{\partial w_i}$

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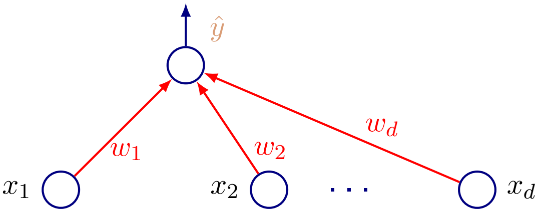

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$$\frac{\partial L}{\partial w_i} =$$

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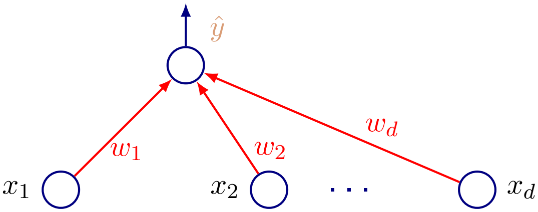

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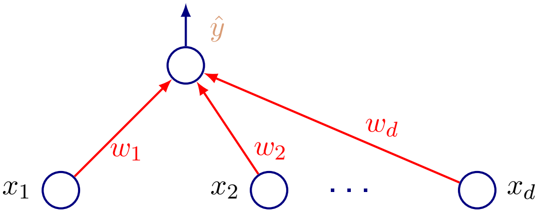

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$$\frac{\partial L}{\partial w_i} = \frac{\partial (\hat{y} - y)^2}{\partial w_i} = 2(\hat{y} - y) \frac{\partial (x_1 w_1 + x_2 w_2 + \dots x_d w_d)}{\partial w_i}$$

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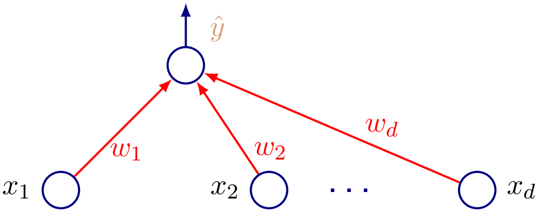
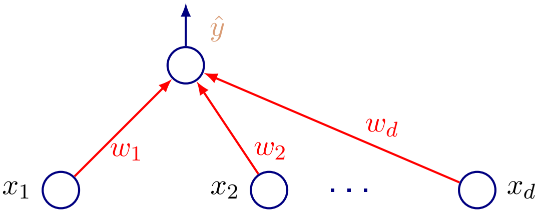


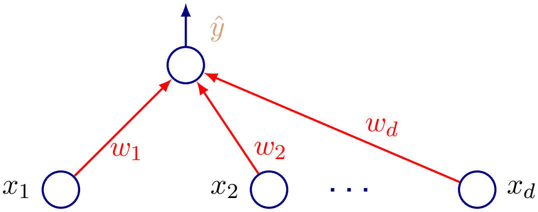
- We have: $\dfrac{\partial L}{\partial w_i} =$

One Neuron Again



- We have: $\frac{\partial L}{\partial w_i} = 2(\hat{y} - y)x_i$

One Neuron Again



- We have: $\dfrac{\partial L}{\partial w_i} = 2(\hat{y} - y)x_i$
- Update Rule:

$$w_i := w_i - \eta(\hat{y} - y)x_i = w_i - \eta\delta x_i \text{ where } \delta = (\hat{y} - y)$$

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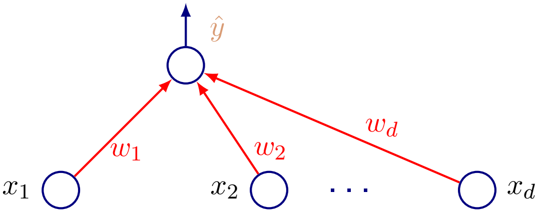

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- In vector form: $\mathbf{w} := \mathbf{w} - \eta\delta\mathbf{x}$

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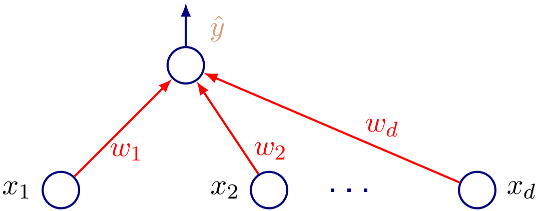

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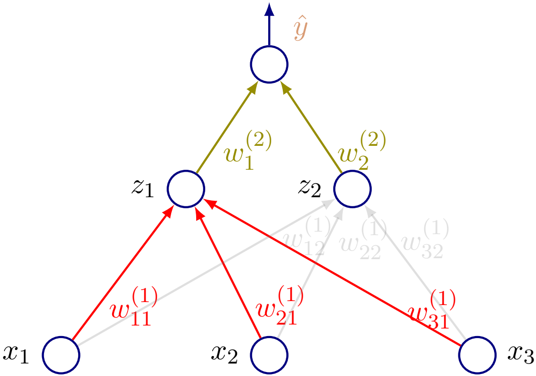
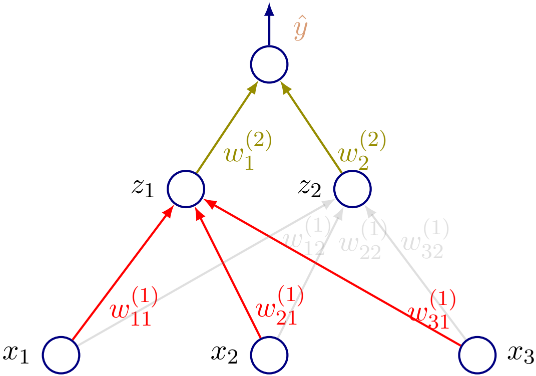
- In vector form: $\mathbf{w} := \mathbf{w} - \eta\delta\mathbf{x}$
- Simple enough! Now let's graduate ...

Simple Feedforward Network



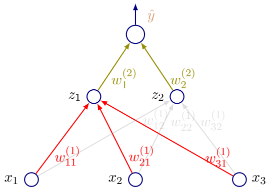
- $\hat{y} = w_1^{(2)} z_1 + w_2^{(2)} z_2$

Simple Feedforward Network



- $\hat{y} = w_1^{(2)} z_1 + w_2^{(2)} z_2$
- $z_1 = \tanh(a_1)$ where $a_1 = w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 + w_{31}^{(1)} x_3$ likewise for z_2

Simple Feedforward Network



- $z_1 = \tanh(a_1)$ where $a_1 = w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 + w_{31}^{(1)} x_3$
- $z_2 = \tanh(a_2)$ where $a_2 = w_{12}^{(1)} x_1 + w_{22}^{(1)} x_2 + w_{32}^{(1)} x_3$

- Output $\hat{y} = w_1^{(2)} z_1 + w_2^{(2)} z_2$; Loss $L = (\hat{y} - y)^2$

Simple Feedforward Network

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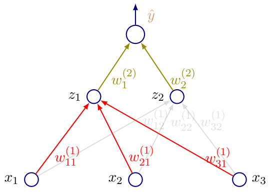
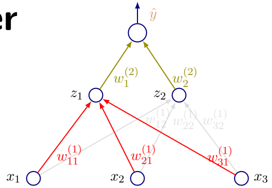


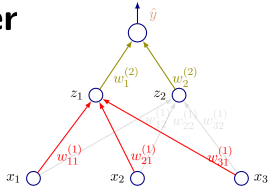
- Output $\hat{y} = w_1^{(2)} z_1 + w_2^{(2)} z_2$; Loss $L = (\hat{y} - y)^2$
- Want to assign credit for the loss L to each weight

Top Layer



- Want to find: $\frac{\partial L}{\partial w_1^{(2)}}$ and $\frac{\partial L}{\partial w_2^{(2)}}$
- Consider $w_1^{(2)}$ first

Top Layer



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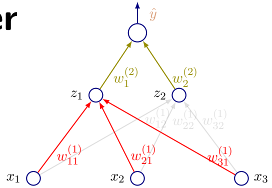

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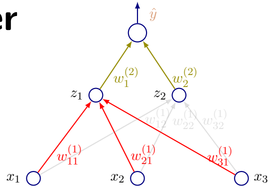

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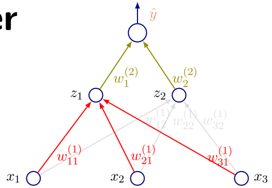

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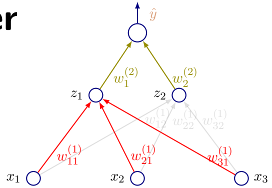

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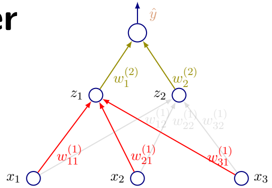
Top Layer



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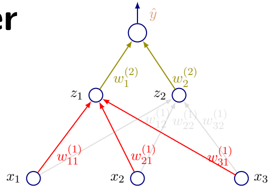

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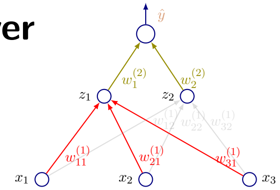
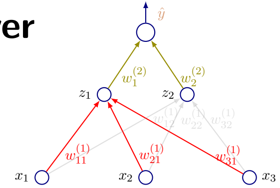
- Likewise, for $w_2^{(2)}$ update would be $w_2^{(2)} := w_2^{(2)} - \eta \delta z_2$

Next Layer



- There are six weights to assign credit for the loss incurred
- Consider $w_{11}^{(1)}$ for an illustration
- Rest are similar

Next Layer



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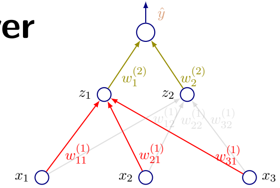
- $\frac{\partial L}{\partial w_{11}^{(1)}} =$

Next Layer



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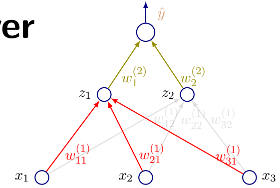
Next Layer



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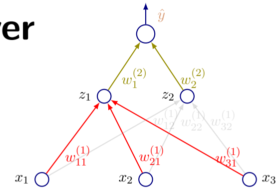
Next Layer



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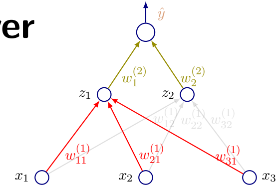
- Now: $\frac{\partial (w_1^{(2)} z_1 + w_2^{(2)} z_2)}{\partial w_{11}^{(1)}} =$

Next Layer



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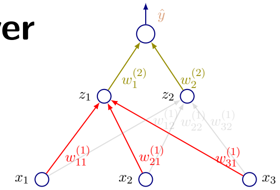
- Now: $\frac{\partial (w_1^{(2)} z_1 + w_2^{(2)} z_2)}{\partial w_{11}^{(1)}} = w_1^{(2)} \frac{\partial (\tanh(w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 + w_{31}^{(1)} x_3))}{\partial w_{11}^{(1)}} + 0$

Next Layer



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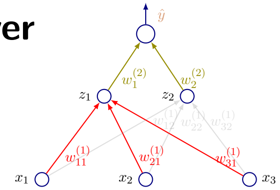
- Which is: $w_1^{(2)}(1-\tanh^2(a_1))x_1$

Next Layer



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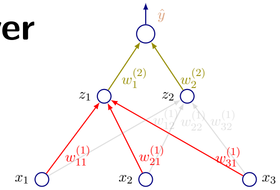
- Which is: $w_1^{(2)}(1-\tanh^2(a_1))x_1$ recall $a_1 =?$

Next Layer



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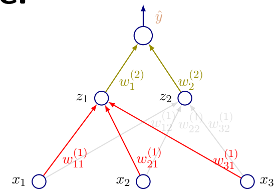
- Which is: $w_1^{(2)}(1-\tanh^2(a_1))x_1$ recall $a_1 = ?$
- So we have: $\frac{\partial L}{\partial w_{11}^{(1)}} = 2(\hat{y}-y) w_1^{(2)}(1-\tanh^2(a_1))x_1$

Next Layer



- $\frac{\partial L}{\partial w_{11}^{(1)}} =$
  $2(\hat{y} - y) w_1^{(2)} (1 - \tanh^2(a_1)) x_1$
- Weight update:
  $w_{11}^{(1)} := w_{11}^{(1)} - \eta \frac{\partial L}{\partial w_{11}^{(1)}}$

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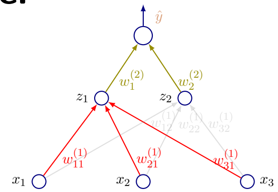

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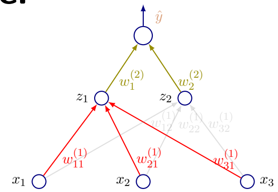
- Likewise, if we were considering $w_{22}^{(1)}$, we'd have:

Next Layer



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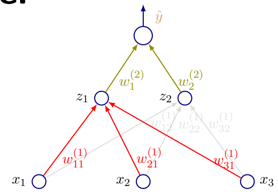
- $\frac{\partial L}{\partial w_{22}^{(1)}} = 2(\hat{y} - y)w_2^{(2)}(1 - \tanh^2(a_2))x_2$

Next Layer



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- Neat!

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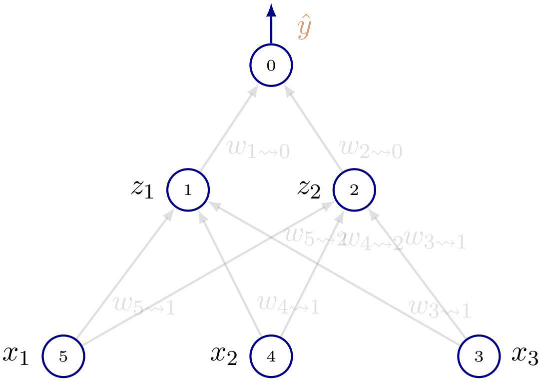
$$\frac{\partial L}{\partial w_{i \rightsquigarrow j}} = \delta_j z_i$$

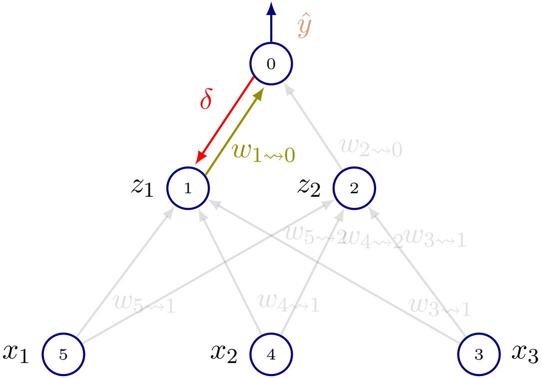
- δ_j is the local error (going from j backwards) and z_i is the local input coming from i

Credit Assignment: A Graphical Revision



- Let's redraw our toy network with new notation and label nodes

Credit Assignment: Top Layer



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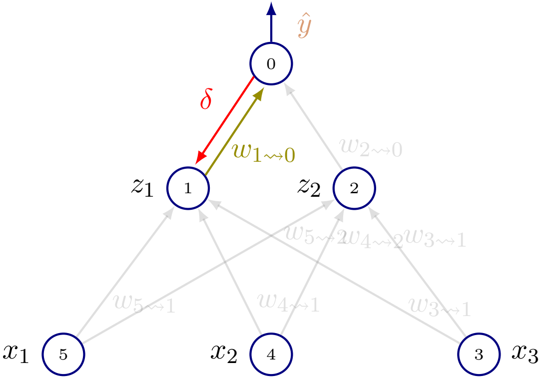

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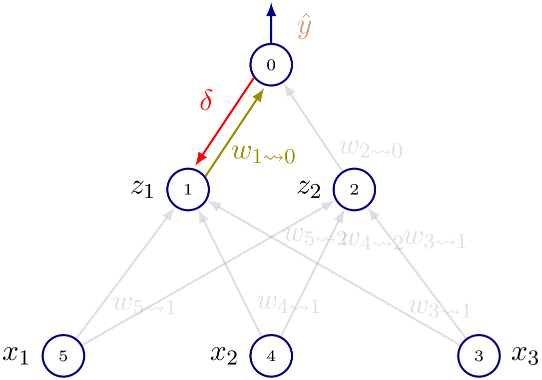

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$$\therefore \frac{\partial L}{\partial w_{1\rightsquigarrow 0}} = \delta z_1; \text{ and update } w_{1\rightsquigarrow 0} := w_{1\rightsquigarrow 0} - \eta \delta z_1$$

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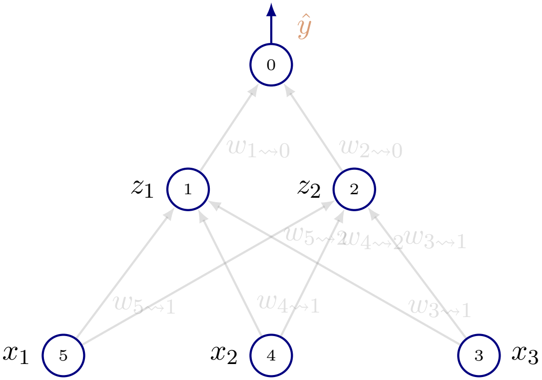

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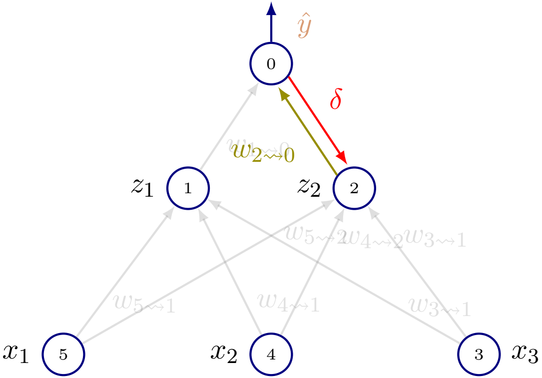

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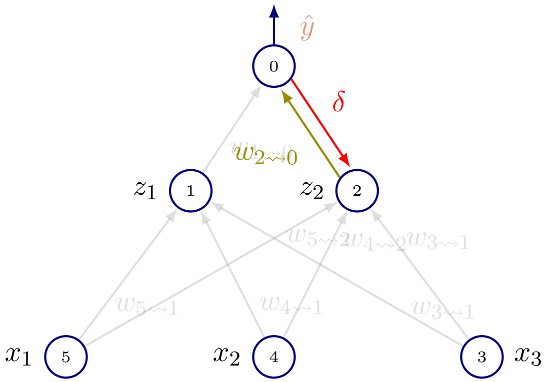

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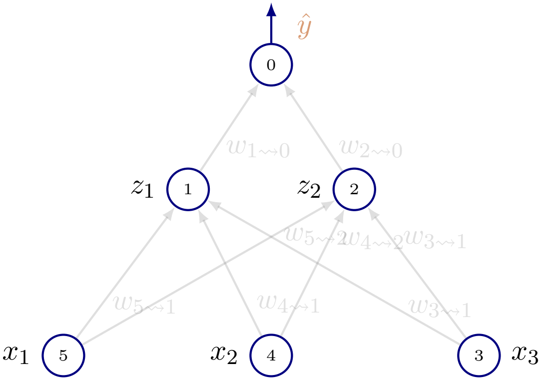

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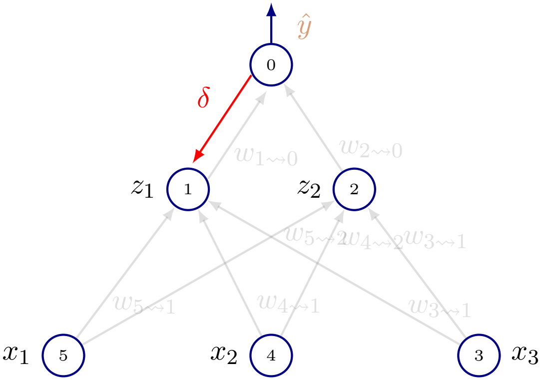

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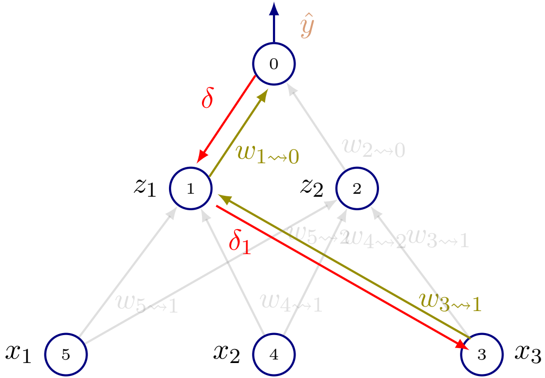

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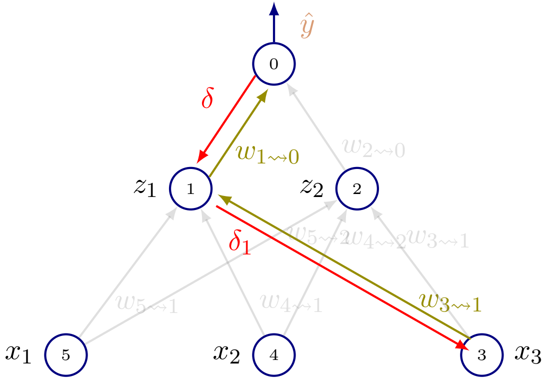

- Local error from 1: $\delta_1 = (\delta)(w_{1\rightsquigarrow 0})(1 - \tanh^2(a_1))$, local input from 3: x_3

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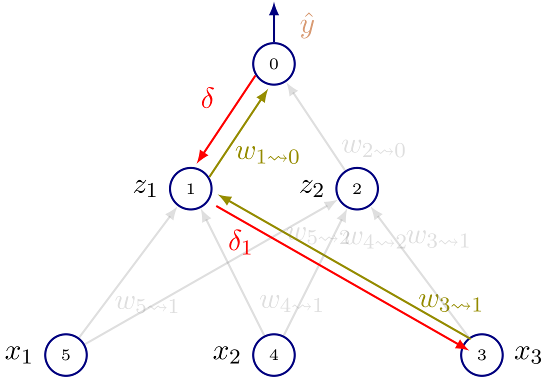

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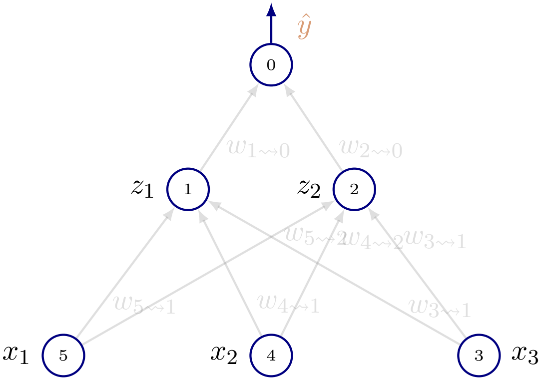

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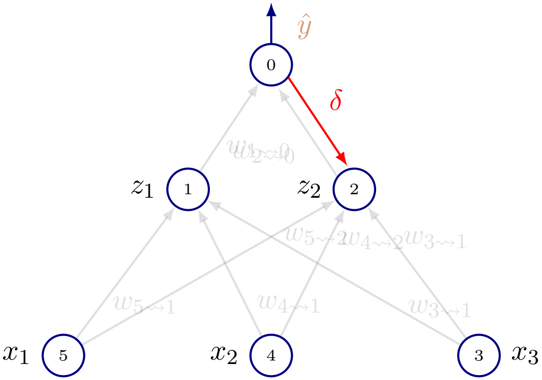

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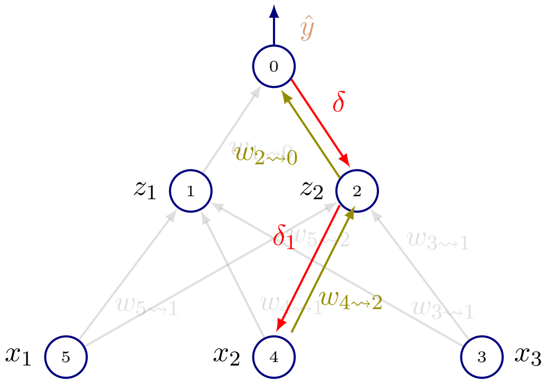

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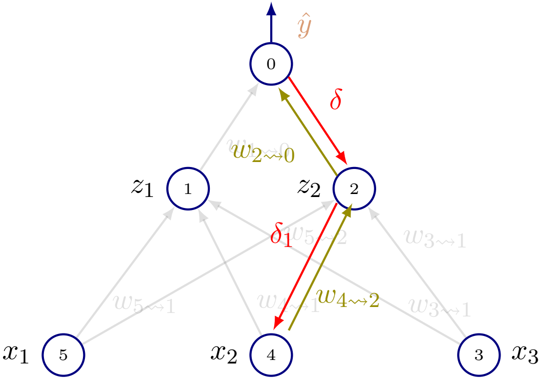
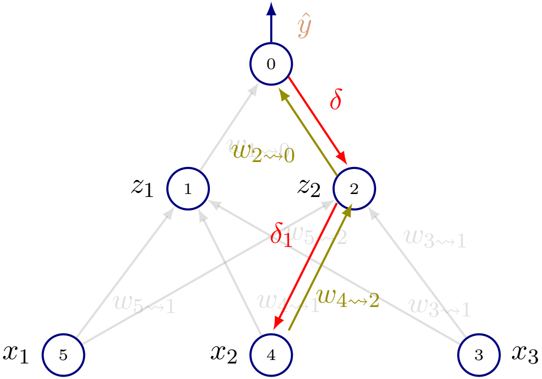


- Local error from 2: $\delta_2 = (\delta)(w_{2\rightsquigarrow 0})(1 - \tanh^2(a_2))$, local input from 4: x_2

Credit Assignment: Next Layer



- Local error from 2: $\delta_2 = (\delta)(w_{2\rightsquigarrow 0})(1 - \tanh^2(a_2))$, local input from 4: x_2

$$\therefore \frac{\partial L}{\partial w_{4\rightsquigarrow 2}} = \delta_2 x_2 \text{ and update } w_{4\rightsquigarrow 2} := w_{4\rightsquigarrow 2} - \eta \delta_2 x_2$$

Let's Vectorize

- Let $W^{(2)} = \begin{bmatrix} w_{1\rightsquigarrow 0} \\ w_{2\rightsquigarrow 0} \end{bmatrix}$ (ignore that $W^{(2)}$ is a vector and hence more appropriate to use $\mathbf{w}^{(2)}$)

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- Let

$$Z^{(1)} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } Z^{(2)} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

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7. All the dimensionalities nicely check out!

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  - We find the δs from the top to the edge concerned by using the chain rule
  - Once we have the partial derivative, we can write the update rule for that weight

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- Need to book-keep derivatives as we go down the network and reuse them

A General View of Backpropagation

Some redundancy in upcoming slides, but redundancy can be good!

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- First: Move to more precise computational graph language!

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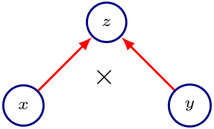
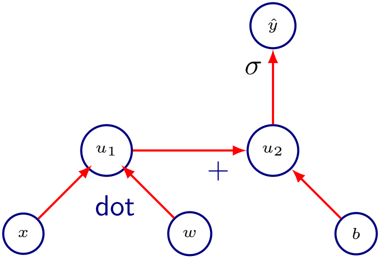
- Examples:

$$z = xy$$



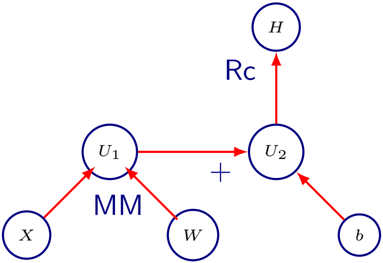
- Graph uses $\times$ operation for the computation

Logistic Regression



- Computes $\hat{y} = \sigma(\mathbf{x}^T\mathbf{w} + b)$

$$H = \max\{0, XW + b\}$$



MM is matrix multiplication and Rc is ReLU activation

Back to backprop: Chain Rule

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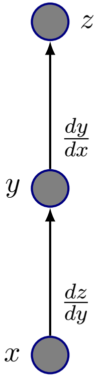
- Suppose $y = g(x)$ and $z = f(y) = f(g(x))$
- Chain rule:

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$



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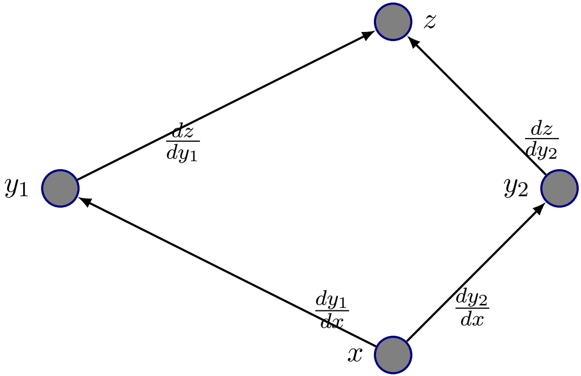
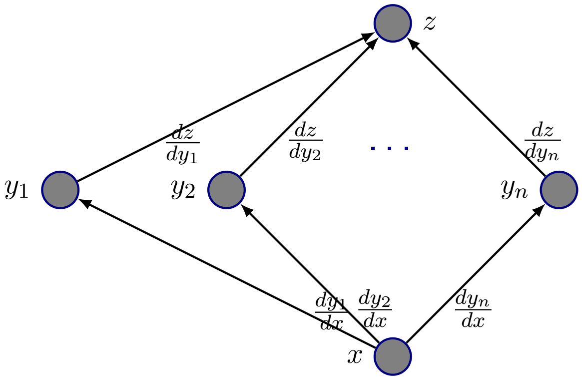


Multiple Paths: $\frac{dz}{dx} = \frac{dz}{dy_1}\frac{dy_1}{dx} + \frac{dz}{dy_2}\frac{dy_2}{dx}$



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- In vector notation:

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- In general this need not only apply to vectors, but can apply to tensors w.l.o.g

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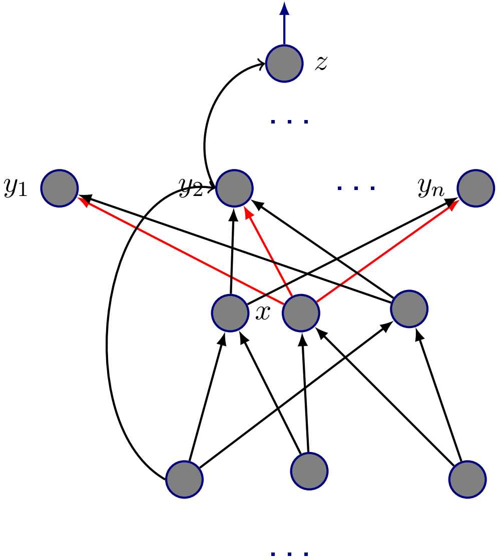
Flow Graph (for previous slide)



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- Makes for rapid prototyping

Computational Graph for a MLP

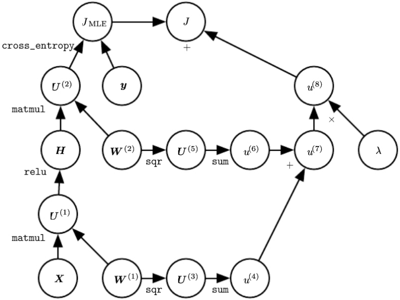

Figure: Goodfellow *et al.*

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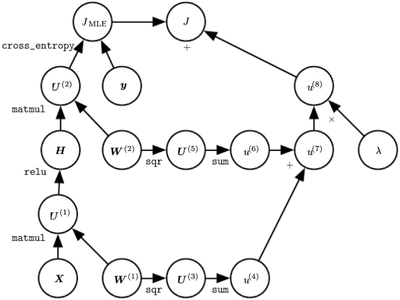

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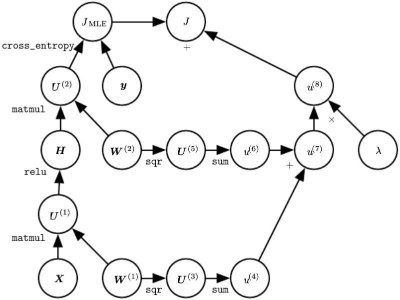

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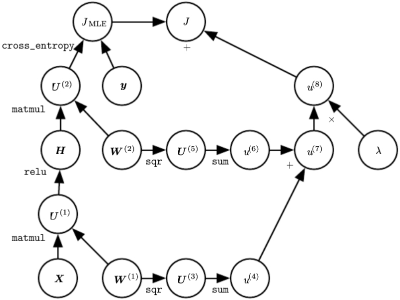

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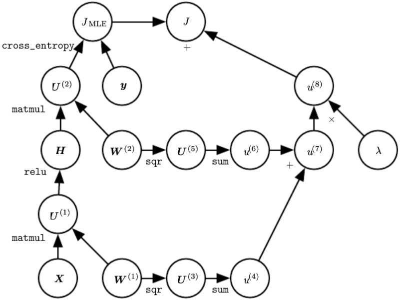

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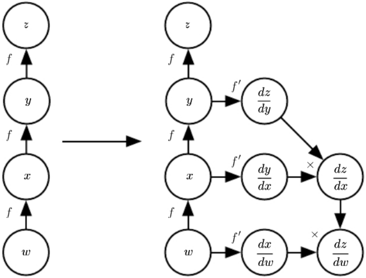

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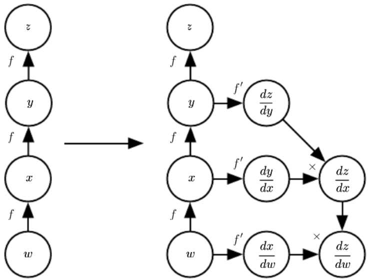

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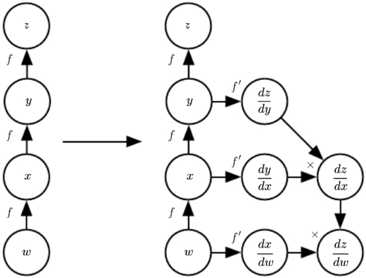

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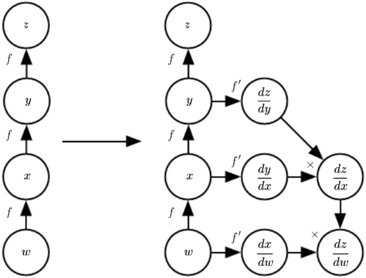

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- Approach taken by Theano and TensorFlow

Next time

- Regularization Methods for Deep Neural Networks