Lecture 11
Recurrent Neural Networks I
CMSC 35246: Deep Learning

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May 01, 2017

Introduction

Sequence Learning with Neural Networks

Some Sequence Tasks

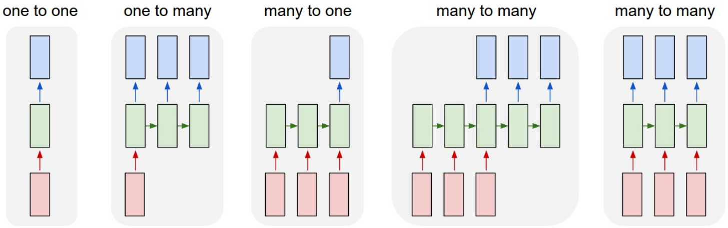

Figure credit: Andrej Karpathy

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- Need to resuse knowledge about the previous events to help in classifying the current.

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- Recurrent networks share parameters: Each output is a function of the previous outputs, with the same update rule applied

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- This expression does not involve any recurrence and can be represented by a traditional directed acyclic computational graph

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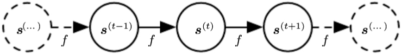

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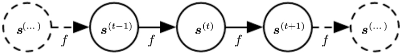

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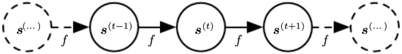

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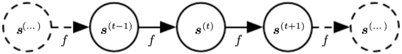

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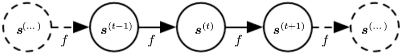

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- RNNs can be built in various ways: Just as any function can be considered a feedforward network, any function involving a recurrence can be considered a recurrent neural network

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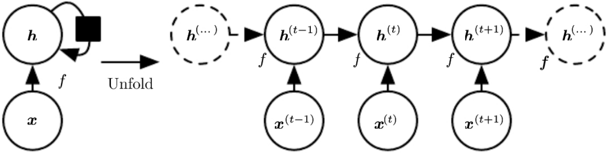

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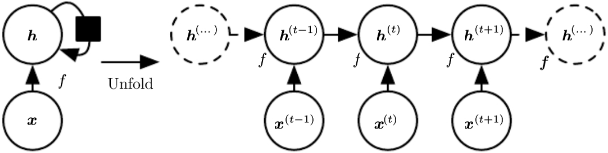

- We can have additional architectural features: Such as output layers that read information from h to make predictions

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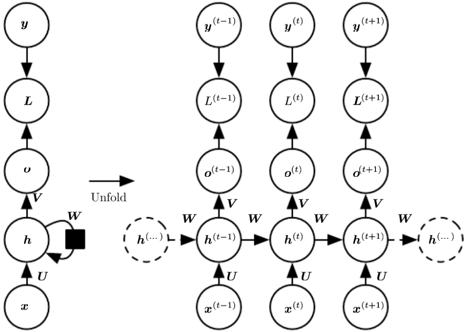
- Most demanding situation for $h^{(t)}$: Approximately recover the input sequence

Design Patterns of Recurrent Networks



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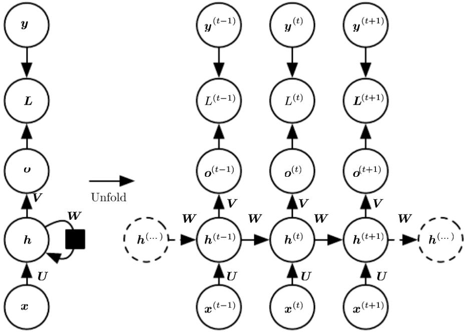

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- Is infact Turing Complete (Siegelmann, 1991, 1995, 1995)

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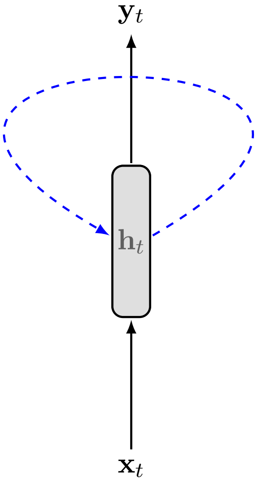
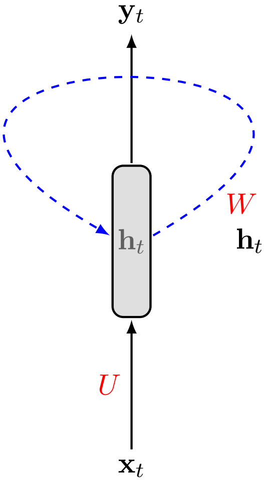
Plain Vanilla Recurrent Network



Recurrent Connections



$$\mathbf{h}_t = \psi(U\mathbf{x}_t + W\mathbf{h}_{t-1})$$

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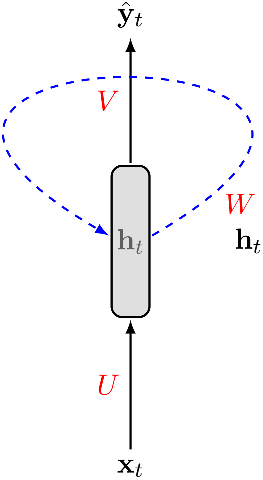

$$\hat{\mathbf{y}}_t = \phi(V\mathbf{h}_t)$$

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ψ can be $\tanh$ and ϕ can be softmax

Unrolling the Recurrence

$\mathbf{x}_1 \qquad \mathbf{x}_2 \qquad \mathbf{x}_3 \qquad \cdots \qquad \mathbf{x}_\tau$

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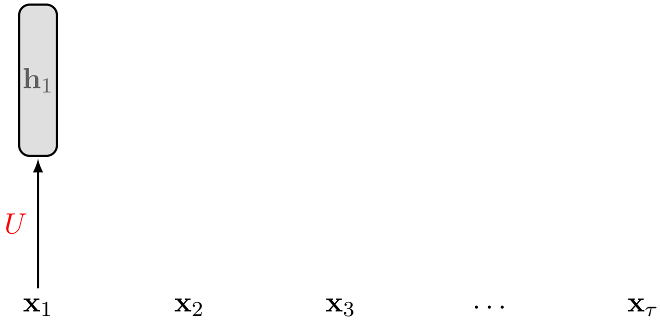

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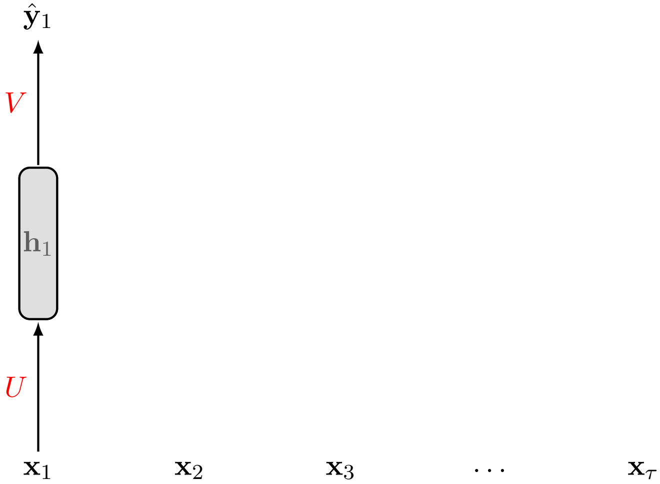

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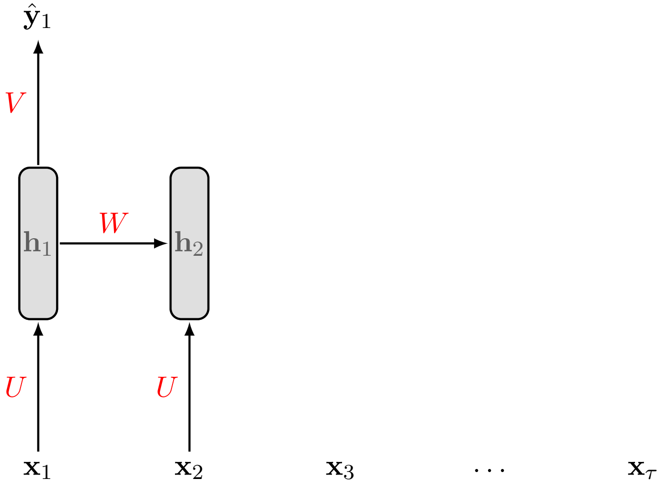

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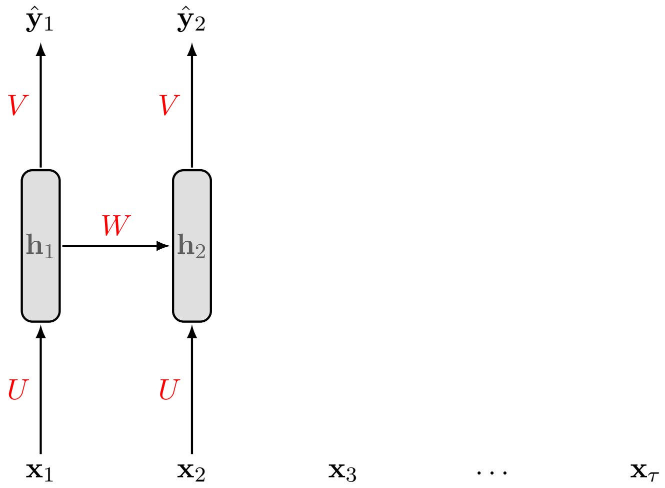

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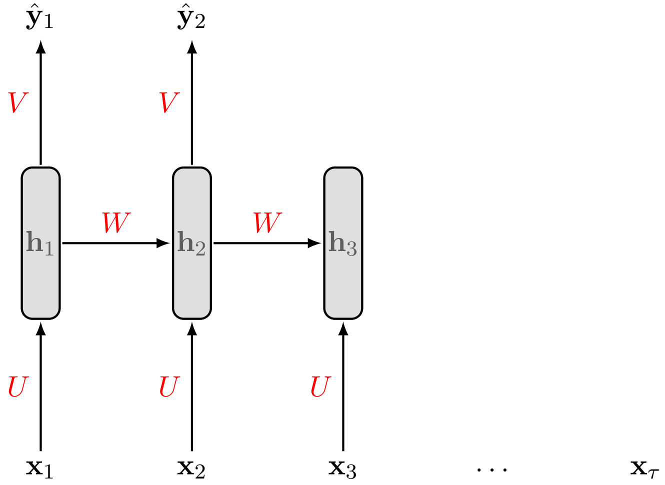

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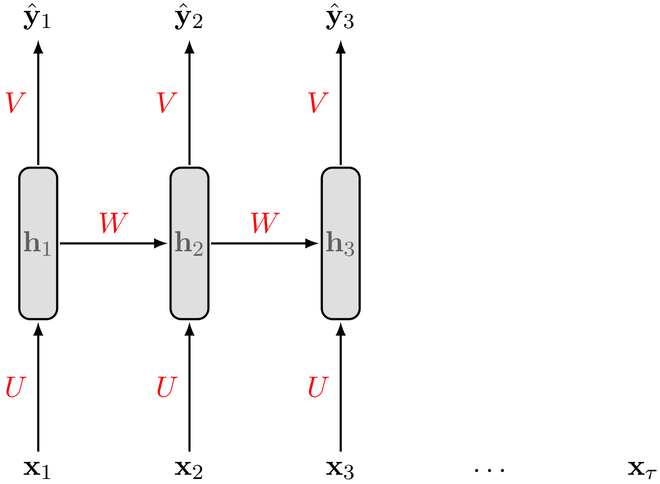

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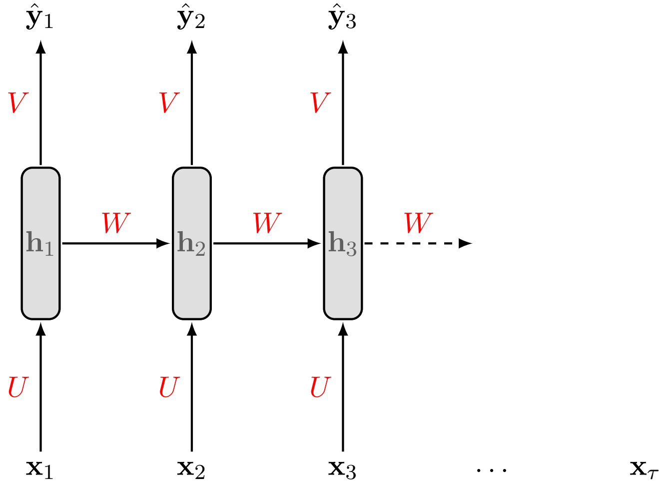

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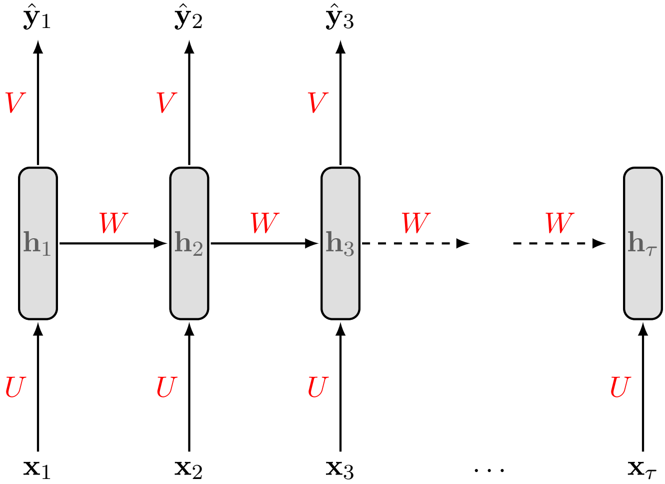

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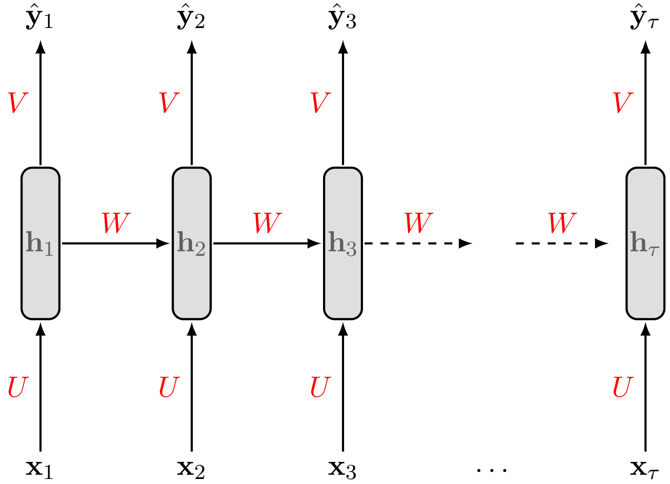

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$$\begin{aligned}\mathbf{a}_t &= b + W\mathbf{h}_{t-1} + U\mathbf{x}_t \\ \mathbf{h}_t &= \tanh \mathbf{a}_t \\ \mathbf{o}_t &= c + V\mathbf{h}_t \\ \hat{\mathbf{y}}_t &= \text{softmax}(\mathbf{o}_t)\end{aligned}$$

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- Observation: Forward propagation takes time $O(t)$; can't be parallelized

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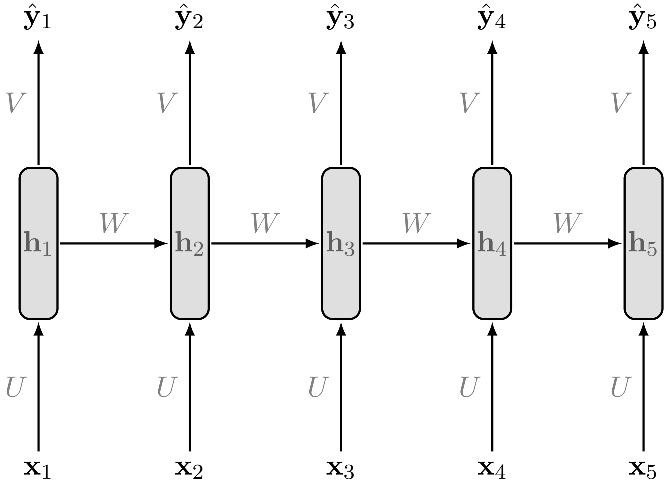
BPTT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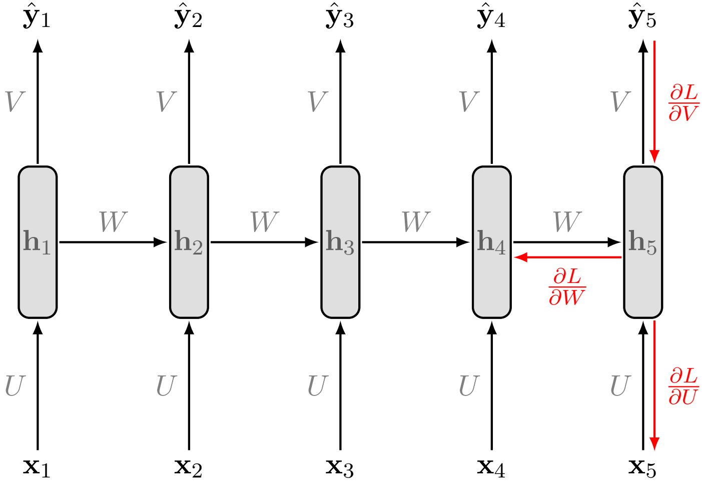
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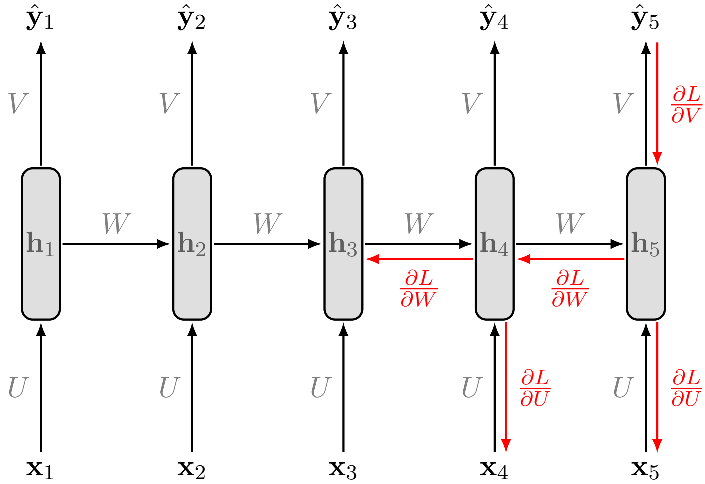

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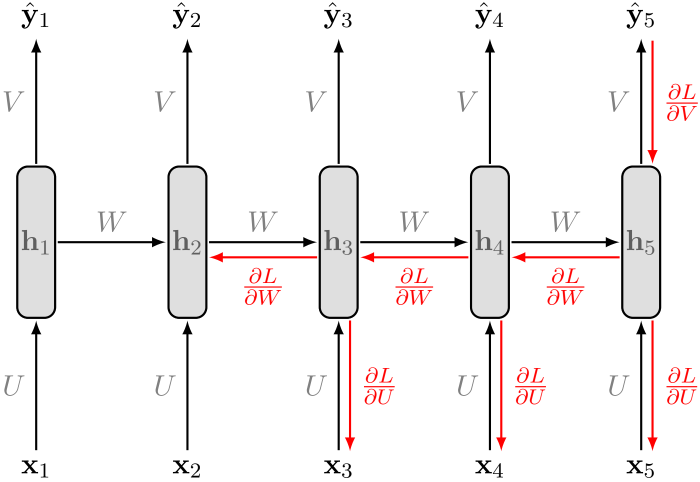
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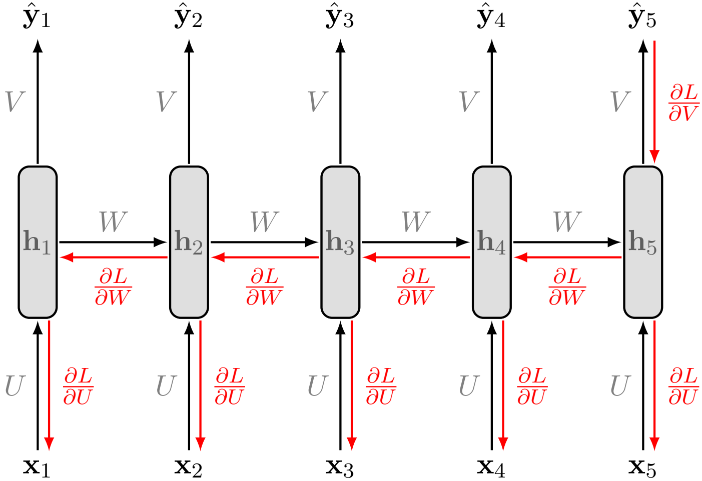
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U
$\frac{\partial L}{\partial U}$
$\mathbf{x}_1$
$\mathbf{x}_2$
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Gradient Computation

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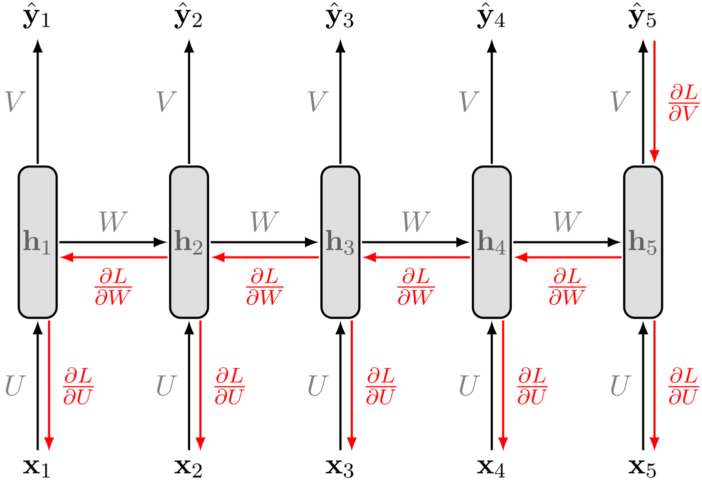
Where:

$$(\nabla_{\mathbf{o}_t} L)_i = \frac{\partial L}{\partial \mathbf{o}_t^{(i)}} = \frac{\partial L}{\partial L_t}\frac{\partial L_t}{\partial \mathbf{o}_t^{(i)}} = \hat{\mathbf{y}}_t^{(i)} - \mathbf{1}_{i,\mathbf{y}_t}$$

BPTT


$\hat{\mathbf{y}}_1$
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Gradient Computation

$$\nabla_W L = \sum_t diag\big(1 - (\mathbf{h}_t)^2\big)(\nabla_{\mathbf{h}_t} L)\mathbf{h}_{t-1}^T$$

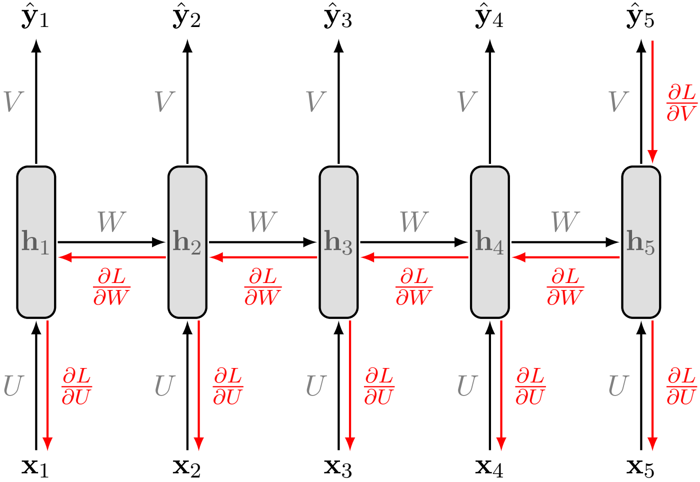
Where, for $t = \tau$ (one descendant):

$$(\nabla_{\mathbf{h}_\tau} L) = V^T (\nabla_{\mathbf{o}_\tau} L)$$

For some $t < \tau$ (two descendants)

$$\begin{aligned}(\nabla_{\mathbf{h}_t} L) &= \Big(\frac{\partial \mathbf{h}_{t+1}}{\partial \mathbf{h}_t}\Big)^T (\nabla_{\mathbf{h}_{t+1}} L) + \Big(\frac{\partial \mathbf{o}_t}{\partial \mathbf{h}_t}\Big)^T (\nabla_{\mathbf{o}_t} L) \\ &= W^T (\nabla_{h_{t+1}} L) diag(1 - \mathbf{h_{t+1}}^2) + V(\nabla_{\mathbf{o}_t} L)\end{aligned}$$

BPTT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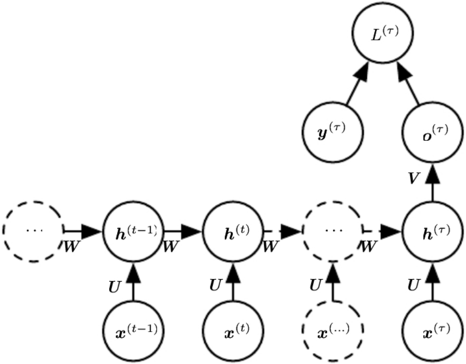
Recurrent Neural Networks

- But weights are shared across different time stamps? How is this constraint enforced?

Recurrent Neural Networks

- But weights are shared across different time stamps? How is this constraint enforced?
- Train the network as if there were no constraints, obtain weights at different time stamps, average them

Design Patterns of Recurrent Networks



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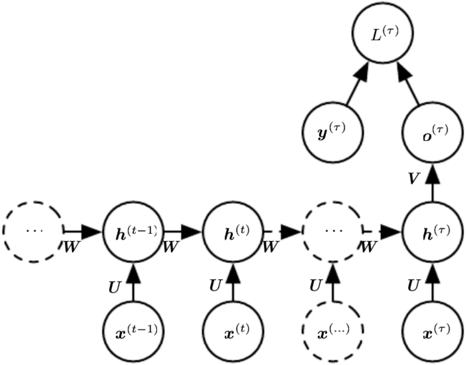

- **Summarization:** Produce a single output and have recurrent connections from output between hidden units
- Useful for summarizing a sequence (e.g. sentiment analysis)

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- We have considered RNNs in the context of a sequence of vectors $x^{(t)}$ with $t = 1, \ldots, \tau$ as input

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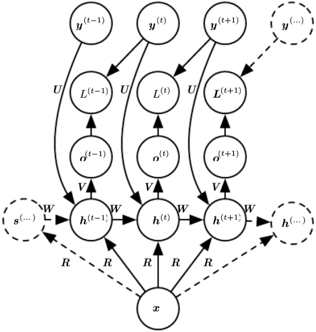

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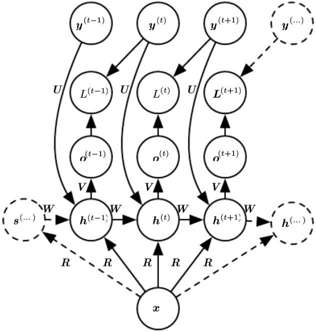

- Maps a fixed vector x into a distribution over sequences Y ($x^T R$ effectively is a new bias parameter for each hidden unit)

Application: Caption Generation

man in black shirt is playing guitar.

construction worker in orange safety

two young girls are playing with lego

boy is doing backflip on wakeboard.

Caption Generation

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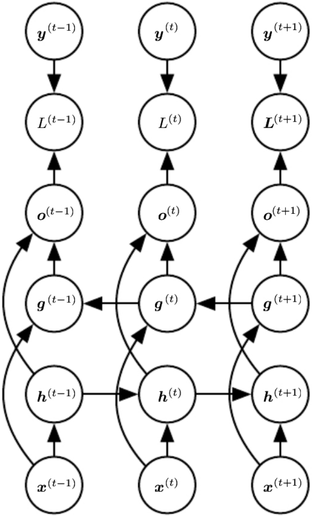
- Basically, in many cases we are interested in looking into the future as well as the past to disambiguate interpretations
- Bidirectional RNNs were introduced to address this need (Schuster and Paliwal, 1997), and have been used in handwriting recognition (Graves 2012, Graves and Schmidhuber 2009), speech recognition (Graves and Schmidhuber 2005) and bioinformatics (Baldi 1999)

Design Patterns: Bidirectional RNNs



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- C could be a vector or a sequence that summarizes $X = \{x^{(1)}, \ldots, x^{(n_x)}\}$

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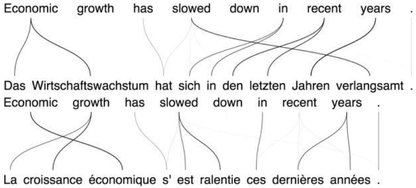
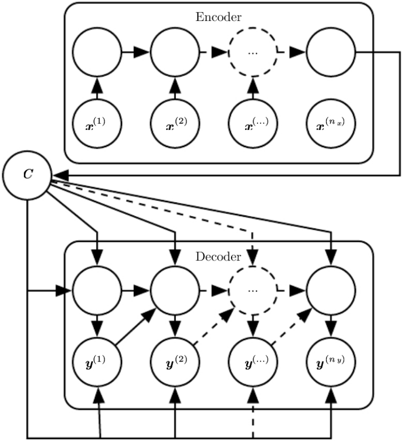


- Far more complicated mappings

Design Patterns: Encoder-Decoder



- In the context of Machine Trans. C is called a thought vector

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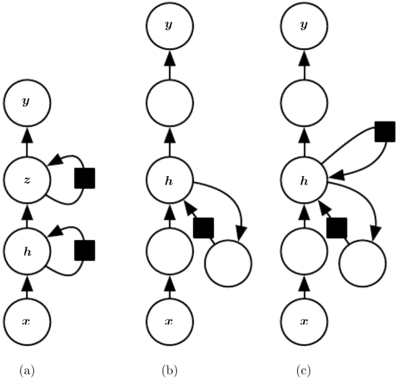
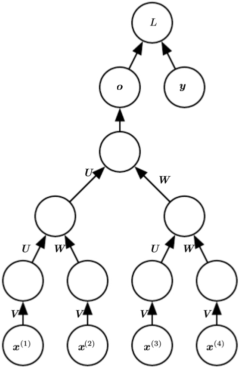
Deep Recurrent Networks



(b) lengthens shortest paths linking different time steps, (c) mitigates this by introducing skip layers

Recursive Neural Networks



- The computational graph is structured as a deep tree rather than as a chain in a RNN

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- The computation performed by each node need not be the usual neuron computation - it could instead be tensor operations etc (Socher *et al* 2013)

Long-Term Dependencies

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- The problem was first analyzed by Hochreiter and Schmidhuber 1991 and Bengio *et al* 1993

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- When $|\lambda > 1|$, $\delta|\lambda|^n$ grows exponentially large and vice-versa

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Other Ideas

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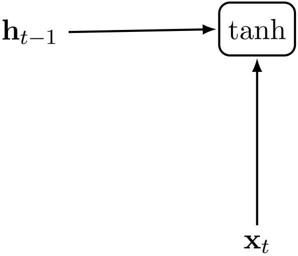
- Leaky Units

Long Short Term Memory



$$\mathbf{h}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$$

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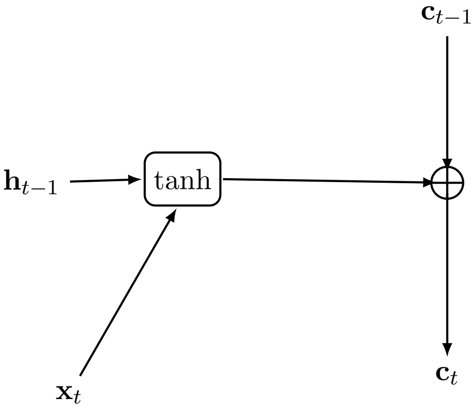
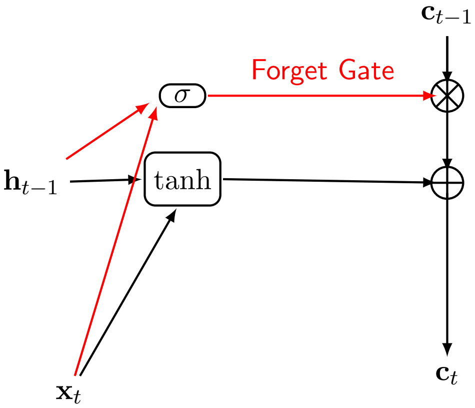


$$\tilde{\mathbf{c}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$$
$$\mathbf{c}_t = \mathbf{c}_{t-1} + \tilde{c}_t$$

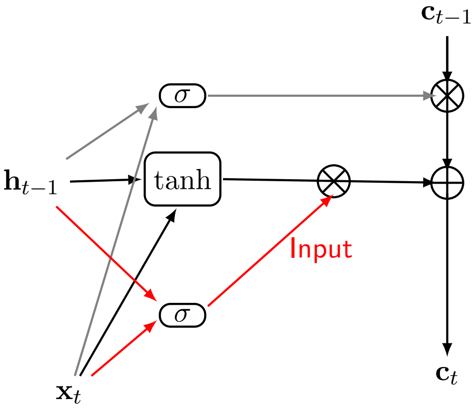
Long Short Term Memory



$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + \tilde{c}_t$$

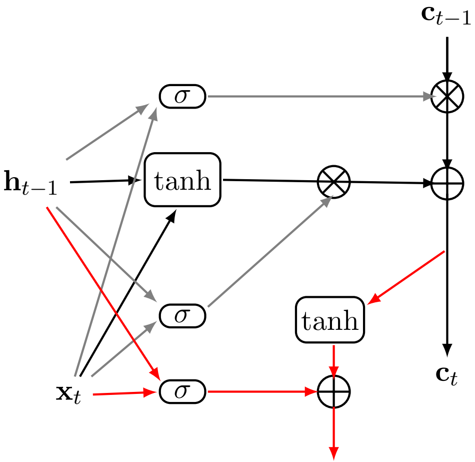
Long Short Term Memory



$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$
$$i_t = \sigma(W_i \mathbf{h}_{t-1} + U_i \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{c}_t$$

Long Short Term Memory



$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$
$$i_t = \sigma(W_i \mathbf{h}_{t-1} + U_i \mathbf{x}_t)$$
$$o_t = \sigma(W_o \mathbf{h}_{t-1} + U_o \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{c}_t$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{c}_t)$$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$
- Reset gate: $r_t = \sigma(W_r\mathbf{h}_{t-1} + U_r\mathbf{x}_t)$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$
- Reset gate: $r_t = \sigma(W_r\mathbf{h}_{t-1} + U_r\mathbf{x}_t)$
- New $\tilde{\mathbf{h}}_t = \tanh(W(r_t \odot \mathbf{h}_{t-1}) + U\mathbf{x}_t)$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$
- Reset gate: $r_t = \sigma(W_r\mathbf{h}_{t-1} + U_r\mathbf{x}_t)$
- New $\tilde{\mathbf{h}}_t = \tanh(W(r_t \odot \mathbf{h}_{t-1}) + U\mathbf{x}_t)$
- Find: $z_t = \sigma(W_z\mathbf{h}_{t-1} + U_z\mathbf{x}_t)$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$
- Reset gate: $r_t = \sigma(W_r\mathbf{h}_{t-1} + U_r\mathbf{x}_t)$
- New $\tilde{\mathbf{h}}_t = \tanh(W(r_t \odot \mathbf{h}_{t-1}) + U\mathbf{x}_t)$
- Find: $z_t = \sigma(W_z\mathbf{h}_{t-1} + U_z\mathbf{x}_t)$
- Update $\mathbf{h}_t = z_t \odot \tilde{\mathbf{h}}_t$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$
- Reset gate: $r_t = \sigma(W_r\mathbf{h}_{t-1} + U_r\mathbf{x}_t)$
- New $\tilde{\mathbf{h}}_t = \tanh(W(r_t \odot \mathbf{h}_{t-1}) + U\mathbf{x}_t)$
- Find: $z_t = \sigma(W_z\mathbf{h}_{t-1} + U_z\mathbf{x}_t)$
- Update $\mathbf{h}_t = z_t \odot \tilde{\mathbf{h}}_t$
- Finally: $\mathbf{h}_t = (1 - z_t) \odot \mathbf{h}_{t-1} + z_t \odot \tilde{\mathbf{h}}_t$

Gated Recurrent Unit

- Let $\tilde{\mathbf{h}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$ and $\mathbf{h}_t = \tilde{\mathbf{h}}_t$
- Reset gate: $r_t = \sigma(W_r\mathbf{h}_{t-1} + U_r\mathbf{x}_t)$
- New $\tilde{\mathbf{h}}_t = \tanh(W(r_t \odot \mathbf{h}_{t-1}) + U\mathbf{x}_t)$
- Find: $z_t = \sigma(W_z\mathbf{h}_{t-1} + U_z\mathbf{x}_t)$
- Update $\mathbf{h}_t = z_t \odot \tilde{\mathbf{h}}_t$
- Finally: $\mathbf{h}_t = (1 - z_t) \odot \mathbf{h}_{t-1} + z_t \odot \tilde{\mathbf{h}}_t$
- Comes from attempting to factor LSTM and reduce gates