Problems for Discussion 2

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Exercise 1.
Consider the vector space V with the set $\mathbb{R}^3_{>0}$ and the operations $+_V$ and $._V$ defined below:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} +_V \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_1 w_1 \\ v_2 w_2 \\ v_3 w_3 \end{pmatrix}$$

$$a \ ._V \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_1^a \\ v_2^a \\ v_3^a \end{pmatrix}$$

(i) What is the zero vector in V?
(ii) Find a basis for V.

Exercise 2.
Let V, W be finite dimensional vector space over some field F, and let φ be a linear map from V to W. Show that there exists a subspace U of V such that (i) $U \cap \ker(\varphi) = \{0_V\}$ (ii) $\mathrm{im}(\varphi) = \{\varphi(u) : u \in U\}$.