

# Practice Problems #6

November 9, 2016

**1)**

Using a “fair” coin, give a method for simulating a single flip of a coin which comes up heads with probability  $1/3$  and tails with probability  $2/3$ . How many flips of the fair coin are needed for your method in expectation?

**2)**

Let  $a$  be a number with binary expansion  $0.b_1b_2b_3\dots$ . Consider the following algorithm:

1. Flip a fair coin until it comes up heads
2. Let  $n$  be the number of flips it took to get a head
3. Return “heads” if  $b_n = 1$  and “tails” otherwise

Prove that this algorithm returns heads with probability  $a$ .

**3)**

Generate the random variable  $X$  using the following algorithm:

1.  $a = 1$
2. Repeat:
3. Flip a fair coin. If it comes up heads, continue, otherwise exit the loop
4.  $a = a * 4/3$
5. EndRepeat:
6. return  $a$

What is  $\mathbb{E}[X]$ ?