Practice Problems

October 26, 2016

Problem 1:

Let the matrix $A : \mathbb{R}^n \mapsto \mathbb{R}^n$ be *strictly diagonally dominant*, meaning that

$$|A_{i,i}| > \sum_{j \neq i} |A_{i,j}|$$

prove that A is non-singular.

Problem 2:

Let $A : \mathbb{R}^n \mapsto \mathbb{R}^n$ be a symmetric, real matrix, and suppose its eigenvalues satisfy the property that $|\lambda_1| > \gamma |\lambda_i| \quad$ for some $\gamma > 1$ and for all $i = 2, ..., n$. Consider the following process:

1. Start with a unit vector $x_0 \in \mathbb{R}^n$
2. For $t = 1, 2, ...$ set $x_t = Ax_{t-1}$

Prove that if $\langle x_0, v_1 \rangle > 0$, where v_1 is the eigenvector corresponding to λ_1, then $\frac{x_t}{\|x_t\|} \to v_1$. This process is called the "power method," which can be used to find the eigenvalues and eigenvectors of A. (Bonus: How could you use this to find eigenvectors besides v_1?)