# Practice Problems for Section 4

October 19, 2016

## 1)

Consider the vector space $V = \mathbb{R}^3_{>0}$ over the field $\mathbb{R}$ with addition

$$[v_1, v_2, v_3] \oplus [w_1, w_2, w_3] = [v_1 w_1, v_2 w_2, v_3 w_3]$$

and scalar multiplication

$$a \otimes [v_1, v_2, v_3] = [v_1^a, v_2^a, v_3^a]$$

Find a basis for $V$.

## 2)

Prove the following statement:
Let $V$ be a vector space with dim(V) = n, let $P : V \to V$ be a linear operator with rank(P) = n, and let $\varphi : V \to V$ be a self-adjoint operator. Then $\varphi$ is positive semidefinite if and only if $P^* \varphi P$ is.

## 3)

Prove half of the Courant Fisher theorem:
Let $\dim(V) = n$ and let $\varphi : V \to V$ be a self-adjoint operator with eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$. Then

$$\lambda_k = \inf_{\substack{S \subseteq V \\ \mathrm{dim(S)=n-k+1}}} \sup_{v \in S \setminus \{0\}} \mathcal{R}_\varphi(v)$$

Hint: find a subset $S$ so that $\lambda_k$ is at most $\sup_{v \in S \setminus \{0\}} \mathcal{R}_\varphi(v)$, then show that for any $S$ it is least that large.