Practice Problems for Section 3

October 12, 2016

1) Consider the vector space $V = \{f \in C^\infty[0,1] : f(0) = f(1)\}$ over $\mathbb{R}$ with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$. What is the adjoint of the derivative operator $\varphi(f) = \frac{df}{dx}$?

2) Consider the vector space $M = \{\text{symmetric } n \times n \text{ real matrices}\}$ over $\mathbb{R}$. For any $A \in M$ let $\lambda_1(A)$ be the largest eigenvalue of A in absolute value. Is $\langle A, B\rangle = |\lambda_1(A)|\,|\lambda_1(B)|$ a valid inner product on M?

3) Let V be a finite-dimensional inner product space over $\mathbb{R}$, and let $\varphi : V \to V$ be a linear operator with adjoint φ^*. Prove that $\text{rank}(\varphi) = \text{rank}(\varphi^*)$.