TTIC 31210: Advanced Natural Language Processing

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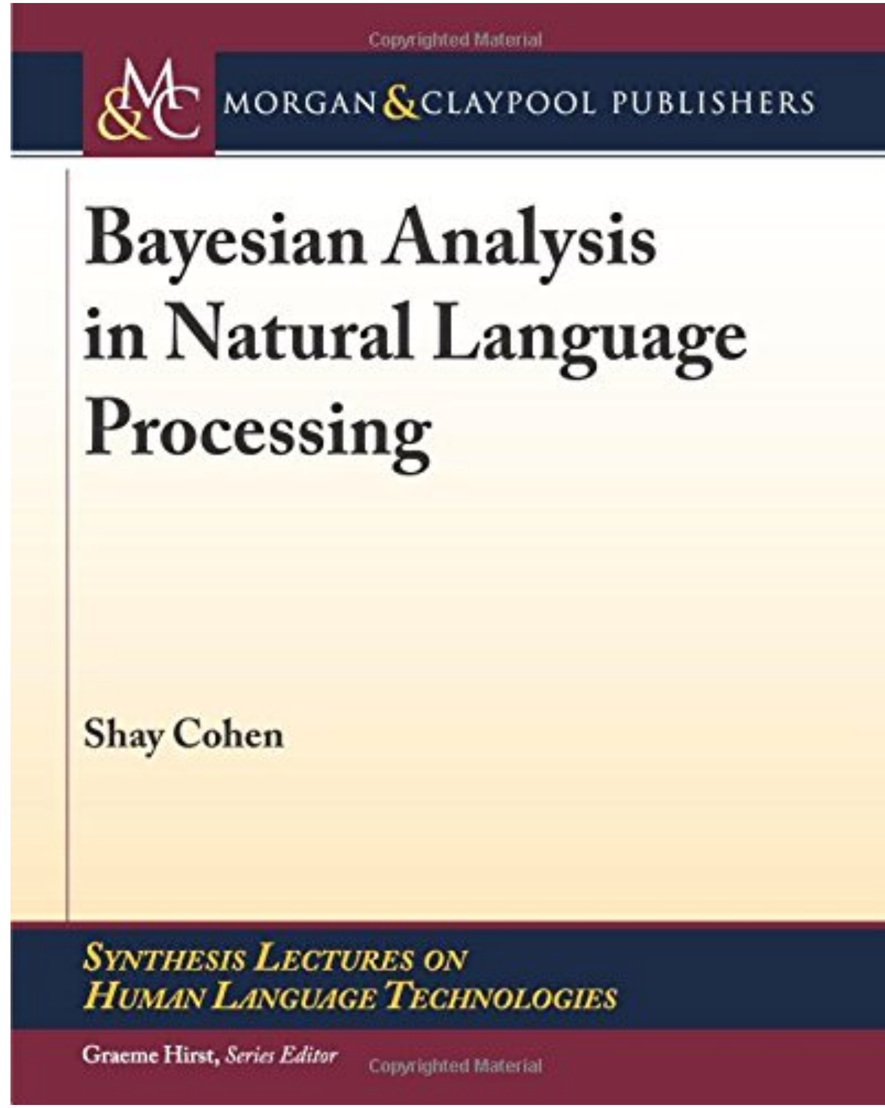
Spring 2017

Lecture 10: Introduction to Bayesian NLP

- Assignment 2 has been posted, due May 17
- Grades for Assignment 1 will be emailed to you soon
- Project proposal details posted, due May 10

Additional Reading

- For this segment of the course, the optional text is Cohen (2016)
- There is a copy in the TTIC library
- Readings will be drawn from this book for the next few lectures



Motivation

- in most neural NLP, we assume parameters and architectures are fixed
- consider a one-hidden-layer MLP:

$$p(Y = y \mid \boldsymbol{x}) = \frac{\exp\{\mathbf{w}_y^\top \tanh\left(\mathbf{W} g(\boldsymbol{x})\right)\}}{Z}$$

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- now let's be more explicit about what we're conditioning on:

$$p(Y = y \mid \boldsymbol{x}, \Theta = \{\mathbf{w}, \mathbf{W}\}) = \frac{\exp\{\mathbf{w}_y^\top \tanh\left(\mathbf{W} g(\boldsymbol{x})\right)\}}{Z}$$

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- marginalize over new random variables:

$$p(Y = y \mid \boldsymbol{x}) = \int_\Theta p(Y = y, \Theta = \{\mathbf{w}, \mathbf{W}\} \mid \boldsymbol{x}) d\Theta$$

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$$p(Y = y, \Theta = \{\mathbf{w}, \mathbf{W}\} \mid \boldsymbol{x}) = p(Y = y \mid \Theta = \{\mathbf{w}, \mathbf{W}\}, \boldsymbol{x})\, p(\Theta = \{\mathbf{w}, \mathbf{W}\} \mid \boldsymbol{x})$$

Going Further…

- marginalize over architectures & parameters:

$$p(Y = y \mid \boldsymbol{x}, \Lambda = \mathrm{MLP}(\mathbf{w}, \mathbf{W})) = \frac{\exp\{\mathbf{w}_y^\top \tanh\left(\mathbf{W} g(\boldsymbol{x})\right)\}}{Z}$$

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- the Bayesian framework gives us a vocabulary to discuss this kind of thing and methods for approximating these computations

Why "Bayesian"?

Likelihood
Probability of collecting this data when our hypothesis is true

Bill Howe, UW

Prior
The probability of the hypothesis being true before collecting data

$$P(H|D) = \frac{P(D|H)\,P(H)}{P(D)}$$

Posterior
The probability of our hypothesis being true given the data collected

Marginal
What is the probability of collecting this data under all possible hypotheses?

Bayesian NLP

- typically used with unsupervised learning:
  - we have data
  - we hypothesize some latent variables through which the data are generated
  - we define the “generative story” that describes how latent variables are generated, then how data is generated using latent variables
  - we parameterize the distributions & add the parameters themselves to the generative story

Generative Story Template

1: Draw a set of parameters θ from $p(\Theta)$

2: Draw a latent structure z from $p(Z \mid \theta)$

3: Draw the observed data x from $p(X \mid z, \theta)$

$$p(x, z, \theta) = p(\theta)p(z \mid \theta)p(x \mid z, \theta)$$

Multinomial Distribution

- parameterized by a vector of probabilities, one for drawing each outcome
- i.e., prob. of drawing outcome i for variable A:

$$p(A = a_i \mid \theta) = \theta_i$$

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$$p(A = a_i \mid \theta) = \theta_i$$

- when we want to draw from this distribution, we will write:

$$a \sim \text{Multinomial}(\theta)$$

- we should more accurately call this a "categorical distribution"
- a multinomial is more general (permits more than 1 instance of an event)
- but multinomial is used frequently to mean categorical in this literature, so we'll often use multinomial

Latent Dirichlet Allocation

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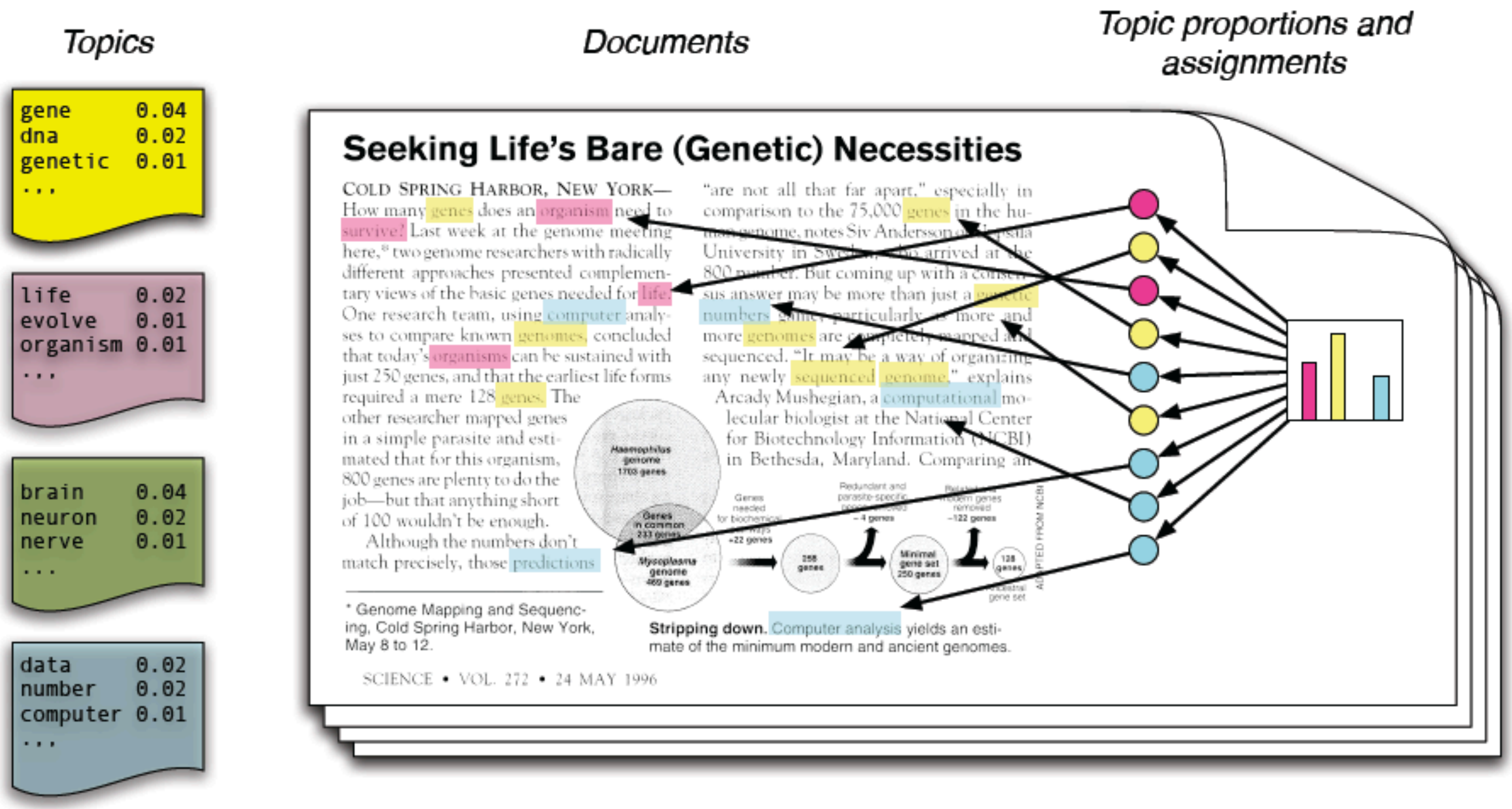
- generative model for document collections using latent variables that can be interpreted as "topics"
- learns a multinomial distribution over words for each topic

Latent Dirichlet Allocation

(Blei et al., 2003)

multinomial distributions over words for four topics:

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT



Topics
Documents
Topic proportions and assignments
gene 0.04
dna 0.02
genetic 0.01
...
life 0.02
evolve 0.01
organism 0.01
...
brain 0.04
neuron 0.02
nerve 0.01
...
data 0.02
number 0.02
computer 0.01
...
Seeking Life's Bare (Genetic) Necessities
* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.
Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.
SCIENCE • VOL. 272 • 24 MAY 1996

Generative Story for Simple LDA

- simplified LDA model, and only showing generative story for a single document:

1: Draw a multinomial topic distribution θ from some distribution $p(\Theta)$

2: For each position i in document:

a: Draw a topic $z_i \sim \text{Multinomial}(\theta)$

b: Draw a word $w_i \sim \text{Multinomial}(\beta_{z_i})$

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multinomial distribution over words for topic z_i

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what should we keep in mind when choosing this distribution?

Dirichlet Distribution

- distribution over vectors with entries that are all positive and sum to 1
- so it's kind of like a "distribution over (multinomial) distributions"

$$p(\Theta = \theta \mid \alpha) = \frac{1}{B(\alpha)} \prod_i \theta_i^{\alpha_i - 1}$$

normalization term that depends on α

Dirichlet Distribution

- parameterized by a positive vector α

$$p(\Theta = \theta \mid \alpha) = \frac{1}{B(\alpha)} \prod_i \theta_i^{\alpha_i - 1}$$

$$\theta \sim \mathrm{Dirichlet}(\alpha)$$

[see Jupyter Notebook]

Generative Story for Simple LDA

- simplified LDA model, and only showing generative story for a single document:

1: Draw a multinomial topic distribution $\theta \sim \text{Dirichlet}(\alpha)$

2: For each position i in document:

a: Draw a topic $z_i \sim \text{Multinomial}(\theta)$

b: Draw a word $w_i \sim \text{Multinomial}(\beta_{z_i})$

Generative Story for LDA

1: For each topic, draw a multinomial word distribution $\beta_i \sim \text{Dirichlet}(\eta)$

2: For each document d:

a: Draw a multinomial topic distribution $\theta \sim \text{Dirichlet}(\alpha)$

b: For each position i in document d:

i: Draw a topic $z_i \sim \text{Multinomial}(\theta)$

ii: Draw a word $w_i \sim \text{Multinomial}(\beta_{z_i})$

- now we show explicitly the generation of the word multinomials (once for the document collection)
- where should the hyperparameters (alpha and psi) come from?

Graphical Model for LDA