# TTIC 31190: <br> Natural Language Processing 

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## Lecture 6: Language Modeling

## Announcements

- Assignment 1 due tonight
- Assignment 2 will be posted today, due Feb. 2
- Midterm scheduled for Thursday, Feb. 18
- Project proposal due Tuesday, Feb. 23
- short (<1 page)
- briefly describe project idea and plan (with timeline)
- one proposal per group (groups can be size 1 or 2)


## Distributional Word Vectors

- simplest way to create word vectors: count occurrences of context words


## Counting Context Words

sugar, a sliced lemon, a tablespoonful of apricot their enjoyment. Cautiously she sampled her first pineapple
well suited to programming on the digital for the purpose of gathering data and information
preserve or jam, a pinch each of, and another fruit whose taste she likened In finding the optimal R-stage policy from necessary for the study authorized in the

|  | aardvark | computer | data | pinch | result | sugar | ... |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| apricot | 0 | 0 | 0 | 1 | 0 | 1 | ... |
| pineapple | 0 | 0 | 0 | 1 | 0 | 1 | ... |
| digital | 0 | 2 | 1 | 0 | 1 | 0 | ... |
| information | 0 | 1 | 6 | 0 | 4 | 0 |  |

## Word-Context Matrix

- assume a vocabulary $V$ and a context vocabulary $V_{C}\left(V_{C}\right.$ is a subset of $\left.V\right)$
- build the word-context matrix $C$
$-C$ is a $|V|-$ by $-\left|V_{C}\right|$ matrix of nonnegative counts
- entry ( $i, j$ ) contains the number of times context word $j$ appeared within $w$ words of word $i$ in a corpus
- then build the PMI matrix $P$


## Pointwise Mutual Information (PMI)

- do two events $x$ and $y$ co-occur more often than if they were independent?

$$
\operatorname{pmi}(x ; y)=\log \frac{p(x, y)}{p(x) p(y)}
$$

- here, $x$ is the center word and $y$ is the word in the context window
- each probability can be estimated from counts collected from a corpus


## Computing PMI


we start with the word-context count matrix $C$ :
$C_{i j}=$ number of times context word $j$ appears in window of word $i$

## Computing PMI

$$
\operatorname{pmi}(i ; j)=\log \frac{p(i, j)}{p(i) p(j)}
$$

$C_{i j}=$ number of times context word $j$ appears in window of word $i$
estimate of joint probability: $p(i, j)=\frac{C_{i j}}{\sum_{i^{\prime}=1}^{|V|} \sum_{j^{\prime}=1}^{\mid V C} C_{i^{\prime} j^{\prime}}}$
$\begin{array}{r}\text { estimates of center } \\ \text { rd and context word }\end{array} \quad p(i)=\frac{\sum_{j=1}^{\left|V_{C}\right|} C_{i j}}{\sum_{i^{\prime}=1}^{|V|} \sum_{j^{\prime}=1}^{\left|V_{C}\right|} C_{i^{\prime} j^{\prime}}}$ word and context word marginal probabilities:

$$
\begin{aligned}
& p(i)=\frac{\sum_{j=1}^{\left|V_{C}\right|} C_{i j}}{\sum_{i^{\prime}=1}^{|V|} \sum_{j^{\prime}=1}^{\left|V_{i}\right|} C_{i^{\prime} j^{\prime}}} \\
& p(j)=\frac{\sum_{i=1}^{|V|} C_{i j}}{\sum_{i^{\prime}=1}^{|V|} \sum_{j^{\prime}=1}^{\left|V_{C}\right|} C_{i^{\prime} j^{\prime}}}
\end{aligned}
$$

pmi(hong, kong) ___ pmi(hong, then)

$$
<>=?
$$

$$
\begin{array}{cc}
\text { pmi(hong, kong) } & >\text { pmi(hong, then) } \\
7.9 & 0.1
\end{array}
$$

## PMIs (1\% of English Wikipedia, window size = 3)

| word | context word | PMI |
| :---: | :---: | :---: |
| hong | kong | 7.9 |
| neither | nor | 6.9 |
| footballer | plays | 6.0 |
| 1980s | 1970s | 5.3 |
| musician | session | 5.0 |
| benefit | doubt | 4.5 |
| gain | failed | 4.0 |
| five | stars | 3.5 |
| miles | distance | 3.0 |
| prior | unlike | 2.0 |
| position | affairs | 1.0 |
| local | processes | 0.5 |
| fire | less | 0.01 |

PMIs (1\% of English Wikipedia, window size = 10)

| word | context word | PMI |
| :---: | :---: | :---: |
| san | francisco | 5.7 |
| san | diego | 5.7 |
| san | juan | 4.7 |
| san | california | 3.7 |
| san | san | 3.6 |
| san | santa | 3.3 |
| word | context word | PMI |
| down | laid | 3.8 |
| down | shot | 3.0 |
| down | turned | 2.9 |
| down | broken | 2.6 |
| down | step | 2.6 |
| down | shooting | 2.5 |

## Evaluating word vectors

- extrinsic:
- question answering, spell checking, essay grading
- intrinsic:
- correlation between vector similarity and human word similarity judgments
- WordSim353: 353 noun pairs rated 0-10
sim(plane,car)=5.77
- TOEFL multiple-choice vocabulary tests


## Roadmap

- classification
- words
- lexical semantics
- language modeling
- sequence labeling
- syntax and syntactic parsing
- neural network methods in NLP
- semantic compositionality
- semantic parsing
- unsupervised learning
- machine translation and other applications


## Probabilistic Language Models

- Today's goal: assign a probability to a sentence
- Why?
- machine translation:
- $P($ high winds tonite $)>P($ large winds tonite $)$
- spelling correction:
- The office is about fifteen minuets from my house
- $P($ about fifteen minutes from $)>P($ about fifteen minuets from $)$
- speech recognition:
- P(I saw a van) >> P(eyes awe of an)
- summarization, question answering, etc.!


## Automatic Completion



## Automatic Completion




## Probabilistic Language Modeling

- goal: compute the probability of a sequence of words:

$$
P(W)=P\left(w_{1}, w_{2}, w_{3}, w_{4}, w_{5} \ldots w_{n}\right)
$$

- related task: probability of next word:

$$
\mathrm{P}\left(\mathrm{w}_{5} \mid \mathrm{w}_{1}, \mathrm{w}_{2}, \mathrm{w}_{3}, \mathrm{w}_{4}\right)
$$

- a model that computes either of these:

$$
\begin{aligned}
& \mathrm{P}(\mathrm{~W}) \text { or } \mathrm{P}\left(\mathrm{w}_{\mathrm{n}} \mid \mathrm{w}_{1}, \mathrm{w}_{2} \ldots \mathrm{w}_{\mathrm{n}-1}\right) \\
& \text { is called a language model }(\mathrm{LM})
\end{aligned}
$$

## How to compute P(W)

- How to compute this joint probability:
- P(its, water, is, so, transparent, that)
- Intuition: let's rely on the Chain Rule of Probability


## Reminder: Chain Rule

- recall definition of conditional probability:

$$
\mathbf{P}(\mathbf{B} \mid \mathbf{A})=\mathbf{P}(\mathbf{A}, \mathrm{B}) / \mathbf{P}(\mathbf{A}) \quad \text { rewriting: } \mathbf{P}(\mathbf{A}, \mathbf{B})=\mathbf{P}(\mathbf{A}) \mathbf{P}(\mathbf{B} \mid \mathbf{A})
$$

- more variables:

$$
P(A, B, C, D)=P(A) P(B \mid A) P(C \mid A, B) P(D \mid A, B, C)
$$

- in general:
$P\left(x_{1}, x_{2}, x_{3}, \ldots, x_{n}\right)=P\left(x_{1}\right) P\left(x_{2} \mid x_{1}\right) P\left(x_{3} \mid x_{1}, x_{2}\right) \ldots P\left(x_{n} \mid x_{1}, \ldots, x_{n-1}\right)$


# Chain Rule applied to computing joint probability of words in sentence 

$$
P\left(w_{1} w_{2} \ldots w_{n}\right)=\prod_{i} P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right)
$$

$\mathrm{P}($ "its water is so transparent") $=$
$\mathrm{P}($ its $) \times \mathrm{P}($ water $\mid$ its $) \times \mathrm{P}($ is $\mid$ its water $)$
$\times \mathrm{P}($ so | its water is $) \times \mathrm{P}($ transparent | its water is so)

## How to estimate these probabilities

- could we just count and divide?
$P($ the lits water is so transparent that $)=$
Count (its water is so transparent that the)
Count(its water is so transparent that)
- no! too many possible sentences!
- we'll never see enough data for estimating these


## Markov Assumption

- simplifying assumption:
$P($ the lits water is so transparent that $) \approx P($ the $\operatorname{lthat})$
- or maybe:
$P($ the lits water is so transparent that $) \approx P($ the I transparent that $)$


## Markov Assumption

$$
P\left(w_{1} w_{2} \ldots w_{n}\right) \approx \prod_{i} P\left(w_{i} \mid w_{i-k} \ldots w_{i-1}\right)
$$

- i.e., we approximate each component in the product:
$P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right) \approx P\left(w_{i} \mid w_{i-k} \ldots w_{i-1}\right)$


## Simplest case: Unigram model

$$
P\left(w_{1} w_{2} \ldots w_{n}\right) \approx \prod_{i} P\left(w_{i}\right)
$$

automatically generated sentences from a unigram model:
fifth an of futures the an incorporated a a the inflation most dollars quarter in is mass
thrift did eighty said hard 'm july bullish
that or limited the

## Bigram model

condition on the previous word:
$P\left(w_{i} \mid w_{1} w_{2} \ldots w_{i-1}\right) \approx P\left(w_{i} \mid w_{i-1}\right)$
automatically generated sentences from a bigram model:
texaco rose one in this issue is pursuing growth in a boiler house said mr. gurria mexico 's motion control proposal without permission from five hundred fifty five yen
outside new car parking lot of the agreement reached
this would be a record november

## n-gram models

- we can extend to trigrams, 4-grams, 5-grams
- in general this is an insufficient model of language
- because language has long-distance dependencies:
"The computer which I had just put into the machine room on the fifth floor crashed."
- but we can often get away with n-gram models


## Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$
\begin{aligned}
& P\left(w_{i} \mid w_{i-1}\right)=\frac{\operatorname{count}\left(w_{i-1}, w_{i}\right)}{\operatorname{count}\left(w_{i-1}\right)} \\
& P\left(w_{i} \mid w_{i-1}\right)=\frac{c\left(w_{i-1}, w_{i}\right)}{c\left(w_{i-1}\right)}
\end{aligned}
$$

## An example

$$
P\left(w_{i} \mid w_{i-1}\right)=\frac{c\left(w_{i-1}, w_{i}\right)}{c\left(w_{i-1}\right)} \quad \begin{aligned}
& \text { <s I I am Sam </s> } \\
& \text { <s Sam I am </s }> \\
& \text { <s I I do not like green eggs and ham </s }>
\end{aligned}
$$

$$
\begin{array}{lll}
P(\mathrm{I} \mid\langle\mathrm{s}\rangle)=\frac{2}{3}=.67 & P(\mathrm{Sam} \mid\langle\mathrm{s}\rangle)=\frac{1}{3}=.33 & P(\mathrm{am} \mid \mathrm{I})=\frac{2}{3}=.67 \\
P(</ \mathrm{s}\rangle \mid \mathrm{Sam})=\frac{1}{2}=0.5 & P(\mathrm{Sam} \mid \mathrm{am})=\frac{1}{2}=.5 & P(\mathrm{do} \mid \mathrm{I})=\frac{1}{3}=.33
\end{array}
$$

## More examples:

## Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day


## Raw bigram counts

- counts from 9,222 sentences
- e.g., "i want" occurs 827 times

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 686 | 2 | 0 | 6 | 211 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

## Raw bigram probabilities

- normalize by unigram counts:

| i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2533 | 927 | 2417 | 746 | 158 | 1093 | 341 | 278 |

- bigram probabilities:

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 0.002 | 0.33 | 0 | 0.0036 | 0 | 0 | 0 | 0.00079 |
| want | 0.0022 | 0 | 0.66 | 0.0011 | 0.0065 | 0.0065 | 0.0054 | 0.0011 |
| to | 0.00083 | 0 | 0.0017 | 0.28 | 0.00083 | 0 | 0.0025 | 0.087 |
| eat | 0 | 0 | 0.0027 | 0 | 0.021 | 0.0027 | 0.056 | 0 |
| chinese | 0.0063 | 0 | 0 | 0 | 0 | 0.52 | 0.0063 | 0 |
| food | 0.014 | 0 | 0.014 | 0 | 0.00092 | 0.0037 | 0 | 0 |
| lunch | 0.0059 | 0 | 0 | 0 | 0 | 0.0029 | 0 | 0 |
| spend | 0.0036 | 0 | 0.0036 | 0 | 0 | 0 | 0 | 0 |

## Bigram estimates of sentence probabilities

$\mathrm{P}(\langle s\rangle$ | want english food $</ s>)=$
$\mathrm{P}(I \mid<s>)$
$\times \mathrm{P}($ want $\mid I)$
$\times \mathrm{P}($ english | want)
$\times \mathrm{P}($ food | english $)$
$\times \mathrm{P}(</ s>\mid$ food $)$
= . 000031

## Practical Issues

- we do everything in log space
- avoid underflow
- (also adding is faster than multiplying)

$$
\log \left(p_{1} \times p_{2} \times p_{3} \times p_{4}\right)=\log p_{1}+\log p_{2}+\log p_{3}+\log p_{4}
$$

## Language Modeling Toolkits

- SRILM
- http://www.speech.sri.com/projects/srilm/
- KenLM
-https://kheafield.com/code/kenlm/


## Google N-Gram Release, August 2006

All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team
Here at Google Research we have been using word n-gram models for a variety of R\&D projects, times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

## Google N-Gram Release

- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensible 40
- serve as the individual 234


## Google Books Ngram Viewer

## Graph these comma-separated phrases: $\quad$ natural language processing,computational linguistics $\quad \square$ case-insensitive

between 1955 and 2008 from the corpus English $\hat{0}$ with smoothing of 3 . Search lots of books


## Google Books Ngram Viewer




## Evaluation: How good is our model?

- does our language model prefer good sentences to bad ones?
- assign higher probability to "real" or "frequently observed" sentences
- than "ungrammatical" or "rarely observed" sentences?


## Extrinsic evaluation of N -gram models

- best evaluation for comparing models $A$ and $B$
- put each model in a task
- spelling corrector, speech recognizer, MT system
- run the task, get an accuracy for $A$ and for $B$
- how many misspelled words corrected properly
- how many words translated correctly
- compare accuracy for $A$ and $B$


## Difficulty of extrinsic evaluation of N -gram models

- extrinsic evaluation is time-consuming
- days or weeks depending on system
- so, sometimes use intrinsic evaluation: perplexity
- bad approximation
- unless the test data looks just like the training data
- so generally only useful in pilot experiments
- but is helpful to think about


## Intuition of Perplexity

- the Shannon Game:
- how well can we predict the next word?

- unigrams are terrible at this game (why?)
- a better model of a text is one which assigns a higher probability to the word that actually occurs


## Perplexity (PP)

best language model is one that best predicts unseen test set

- gives the highest P(sentence)
perplexity $=$ inverse probability of test $\quad P P(W)=P\left(w_{1} w_{2} \ldots w_{N}\right)^{-\frac{1}{N}}$ set, normalized by number of words:

$$
=\sqrt[N]{\frac{1}{P\left(w_{1} w_{2} \ldots w_{N}\right)}}
$$

chain rule:

$$
\operatorname{PP}(W)=\sqrt[N]{\prod_{i=1}^{N} \frac{1}{P\left(w_{i} \mid w_{1} \ldots w_{i-1}\right)}}
$$

for bigrams:

$$
\operatorname{PP}(W)=\sqrt[N]{\prod_{i=1}^{N} \frac{1}{P\left(w_{i} \mid w_{i-1}\right)}}
$$

minimizing perplexity is the same as maximizing probability

## Perplexity as branching factor

- given a sentence consisting of random digits
- what is the perplexity of this sentence according to a model that assigns probability $1 / 10$ to each digit?

$$
\begin{aligned}
\operatorname{PP}(W) & =P\left(w_{1} w_{2} \ldots w_{N}\right)^{-\frac{1}{N}} \\
& =\left(\frac{1}{10}^{N}\right)^{-\frac{1}{N}} \\
& =\frac{1}{10}^{-1} \\
& =10
\end{aligned}
$$

## Lower perplexity = better model

- train: 38 million words
- test: 1.5 million words

> | >  n-gram order: | unigram | bigram | trigram |
| :---: | :---: | :---: | :---: |
| >  perplexity: | 962 | 170 | 109 |
| >  > |  |  |  |

## Approximating Shakespeare

 -Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live -To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have-Hill he late speaks; or! a more to leg less first you enter king. Follow.
-What means, sir. I confess she? then all sorts, he is trim, captain.
-Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.
-This shall forbid it should be branded, if renown made it empty.
-King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
gram -It cannot be but so.

## Shakespeare as corpus

- 884,647 tokens, 29,066 types
- Shakespeare produced 300,000 bigram types out of 844 million possible bigrams
- $99.96 \%$ of possible bigrams were never seen (have zero entries in the table)
- 4-grams worse: what's coming out looks like Shakespeare because it is Shakespeare


## Wall Street Journal



Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

## The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
- in real life, it often doesn't
- we need to train robust models that generalize!
- one kind of generalization: Zeros!
- things that don't ever occur in the training set
-but occur in the test set


## Zeros

training set:
... denied the allegations
... denied the reports
... denied the claims
... denied the request
$P($ offer $\mid$ denied the $)=0$
test set:
... denied the offer ... denied the loan

## Zero probability bigrams

- test set bigrams with zero probability $\rightarrow$ assign 0 probability to entire test set!
- cannot compute perplexity (can't divide by 0 )!


## Intuition of smoothing (from Dan Klein)

- When we have sparse statistics:

$$
\begin{aligned}
& \mathrm{P}(\mathrm{w} \mid \text { denied the }) \\
& 3 \text { allegations } \\
& 2 \text { reports } \\
& 1 \text { claims } \\
& 1 \text { request } \\
& 7 \text { total }
\end{aligned}
$$



- Steal probability mass to generalize better:

$$
\begin{aligned}
& \mathrm{P}(\mathrm{w} \mid \text { denied the }) \\
& 2.5 \text { allegations } \\
& 1.5 \text { reports } \\
& 0.5 \text { claims } \\
& 0.5 \text { request } \\
& 2 \text { other } \\
& 7 \text { total }
\end{aligned}
$$



## "Add-1" estimation

- also called Laplace smoothing
- pretend we saw each word one more time than we did
- just add 1 to all counts!
- MLE estimate:
- Add-1 estimate:

$$
P_{M L E}\left(w_{i} \mid w_{i-1}\right)=\frac{c\left(w_{i-1}, w_{i}\right)}{c\left(w_{i-1}\right)}
$$

$$
P_{A d d-1}\left(w_{i} \mid w_{i-1}\right)=\frac{c\left(w_{i-1}, w_{i}\right)+1}{c\left(w_{i-1}\right)+V}
$$

## Maximum Likelihood Estimates

- The maximum likelihood estimate
- of some parameter of a model M from a training set T
- maximizes the likelihood of the training set $T$ given the model $M$
- Suppose the word "bagel" occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be "bagel"?
- MLE estimate is $400 / 1,000,000=.0004$
- This may be a bad estimate for some other corpus
- But it is the estimate that makes it most likely that "bagel" will occur 400 times in a million word corpus.


## Berkeley Restaurant Corpus: Laplace smoothed bigram counts

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 6 | 828 | 1 | 10 | 1 | 1 | 1 | 3 |
| want | 3 | 1 | 609 | 2 | 7 | 7 | 6 | 2 |
| to | 3 | 1 | 5 | 687 | 3 | 1 | 7 | 212 |
| eat | 1 | 1 | 3 | 1 | 17 | 3 | 43 | 1 |
| chinese | 2 | 1 | 1 | 1 | 1 | 83 | 2 | 1 |
| food | 16 | 1 | 16 | 1 | 2 | 5 | 1 | 1 |
| lunch | 3 | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| spend | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |

## Laplace-smoothed bigrams

$$
P^{*}\left(w_{n} \mid w_{n-1}\right)=\frac{C\left(w_{n-1} w_{n}\right)+1}{C\left(w_{n-1}\right)+V}
$$

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 0.0015 | 0.21 | 0.00025 | 0.0025 | 0.00025 | 0.00025 | 0.00025 | 0.00075 |
| want | 0.0013 | 0.00042 | 0.26 | 0.00084 | 0.0029 | 0.0029 | 0.0025 | 0.00084 |
| to | 0.00078 | 0.00026 | 0.0013 | 0.18 | 0.00078 | 0.00026 | 0.0018 | 0.055 |
| eat | 0.00046 | 0.00046 | 0.0014 | 0.00046 | 0.0078 | 0.0014 | 0.02 | 0.00046 |
| chinese | 0.0012 | 0.00062 | 0.00062 | 0.00062 | 0.00062 | 0.052 | 0.0012 | 0.00062 |
| food | 0.0063 | 0.00039 | 0.0063 | 0.00039 | 0.00079 | 0.002 | 0.00039 | 0.00039 |
| lunch | 0.0017 | 0.00056 | 0.00056 | 0.00056 | 0.00056 | 0.0011 | 0.00056 | 0.00056 |
| spend | 0.0012 | 0.00058 | 0.0012 | 0.00058 | 0.00058 | 0.00058 | 0.00058 | 0.00058 |

## Reconstituted counts

$$
c^{*}\left(w_{n-1} w_{n}\right)=\frac{\left[C\left(w_{n-1} w_{n}\right)+1\right] \times C\left(w_{n-1}\right)}{C\left(w_{n-1}\right)+V}
$$

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 3.8 | 527 | 0.64 | 6.4 | 0.64 | 0.64 | 0.64 | 1.9 |
| want | 1.2 | 0.39 | 238 | 0.78 | 2.7 | 2.7 | 2.3 | 0.78 |
| to | 1.9 | 0.63 | 3.1 | 430 | 1.9 | 0.63 | 4.4 | 133 |
| eat | 0.34 | 0.34 | 1 | 0.34 | 5.8 | 1 | 15 | 0.34 |
| chinese | 0.2 | 0.098 | 0.098 | 0.098 | 0.098 | 8.2 | 0.2 | 0.098 |
| food | 6.9 | 0.43 | 6.9 | 0.43 | 0.86 | 2.2 | 0.43 | 0.43 |
| lunch | 0.57 | 0.19 | 0.19 | 0.19 | 0.19 | 0.38 | 0.19 | 0.19 |
| spend | 0.32 | 0.16 | 0.32 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |

## Compare with raw bigram counts

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 686 | 2 | 0 | 6 | 211 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |


|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 3.8 | 527 | 0.64 | 6.4 | 0.64 | 0.64 | 0.64 | 1.9 |
| want | 1.2 | 0.39 | 238 | 0.78 | 2.7 | 2.7 | 2.3 | 0.78 |
| to | 1.9 | 0.63 | 3.1 | 430 | 1.9 | 0.63 | 4.4 | 133 |
| eat | 0.34 | 0.34 | 1 | 0.34 | 5.8 | 1 | 15 | 0.34 |
| chinese | 0.2 | 0.098 | 0.098 | 0.098 | 0.098 | 8.2 | 0.2 | 0.098 |
| food | 6.9 | 0.43 | 6.9 | 0.43 | 0.86 | 2.2 | 0.43 | 0.43 |
| lunch | 0.57 | 0.19 | 0.19 | 0.19 | 0.19 | 0.38 | 0.19 | 0.19 |
| spend | 0.32 | 0.16 | 0.32 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |

## Add-1 estimation is a blunt instrument

- so add-1 isn't used for N-grams:
- we'll see better methods
- but add-1 is used to smooth other NLP models
- text classification
- domains where the number of zeros isn't so huge


## Backoff and Interpolation

- sometimes it helps to use less context
- condition on less context for contexts you haven't learned much about
- backoff:
- use trigram if you have good evidence, otherwise bigram, otherwise unigram
- interpolation:
- mixture of unigram, bigram, trigram (etc.) models
- interpolation works better


## Linear Interpolation

- simple interpolation:

$$
\begin{aligned}
\hat{P}\left(w_{n} \mid w_{n-2} w_{n-1}\right)= & \lambda_{1} P\left(w_{n} \mid w_{n-2} w_{n-1}\right) \quad \sum_{i} \lambda_{i}=1 \\
& +\lambda_{2} P\left(w_{n} \mid w_{n-1}\right) \\
& +\lambda_{3} P\left(w_{n}\right)
\end{aligned}
$$

- lambdas are functions of context:

$$
\begin{aligned}
\hat{P}\left(w_{n} \mid w_{n-2} w_{n-1}\right)= & \lambda_{1}\left(w_{n-2}^{n-1}\right) P\left(w_{n} \mid w_{n-2} w_{n-1}\right) \\
& +\lambda_{2}\left(w_{n-2}^{n-1}\right) P\left(w_{n} \mid w_{n-1}\right) \\
& +\lambda_{3}\left(w_{n-2}^{n-1}\right) P\left(w_{n}\right)
\end{aligned}
$$

## How to set the lambdas?

- use a held-out corpus:


## Training Data

## Held-Out

 Data
## Test <br> Data

- choose lambdas to maximize probability of held-out data:
- fix N -gram probabilities (on the training data)
- then search for $\lambda$ s that give largest probability to held-out set:

$$
\log P\left(w_{1} \ldots w_{n} \mid M\left(\lambda_{1} \ldots \lambda_{k}\right)\right)=\sum_{i} \log P_{M\left(\lambda_{1} \ldots \lambda_{k}\right)}\left(w_{i} \mid w_{i-1}\right)
$$

- subtlety: what happens if we use training data to learn $\lambda s$ ?


## Unknown words: open vs. closed vocabulary tasks

- if we know all the words in advance:
- vocabulary $V$ is fixed
- "closed vocabulary" task
- often we don't know this
- out-of-vocabulary (OOV) words
- "open vocabulary" task
- so, create an unknown word token <UNK>
- at training time:
- randomly change some instances of rare words to <UNK>
- then estimate its probabilities like a normal word
- at test time:
- replace OOV words with <UNK>


## Huge web-scale n-grams

- how to deal with, e.g., Google N-gram corpus?
- pruning:
- only store N -grams with count > threshold.
- remove singletons of higher-order n-grams
- entropy-based pruning
- efficiency
- efficient data structures like tries
- bloom filters: approximate language models
- store words as indexes, not strings
- use Huffman coding to fit large numbers of words into 2 bytes
- quantize probabilities (4-8 bits instead of 8-byte float)


## Google trillion word language model



More data is better data...


Impact on size of language model training data (in words) on quality of Arabic-English statistical machine translation system


■AE BLEU[\%]
+weblm =
LM trained on 219B words of
 web data

Google
DTSI/ Service Cognitique Robotique et Interaction
44

## Smoothing for Web-scale N-grams

- "Stupid backoff" (Brants et al., 2007)
- no discounting, just use relative frequencies

$$
S\left(w_{i} \mid w_{i-k+1}^{i-1}\right)=\left\{\begin{array}{c}
\frac{\operatorname{count}\left(w_{i-k+1}^{i}\right)}{\operatorname{count}\left(w_{i-k+1}^{i-1}\right)} \text { if } \operatorname{count}\left(w_{i-k+1}^{i}\right)>0 \\
0.4 S\left(w_{i} \mid w_{i-k+2}^{i-1}\right) \quad \text { otherwise }
\end{array}\right.
$$

$$
S\left(w_{i}\right)=\frac{\operatorname{count}\left(w_{i}\right)}{N}
$$

## N-gram Smoothing Summary

- Add-1 estimation:
- OK for text categorization, not for language modeling
- most commonly used method:
- modified interpolated Kneser-Ney
- for very large N -gram collections like the Web:
- stupid backoff


## Advanced Language Modeling

- discriminative models:
- choose n-gram weights to improve a task, not to fit the training set
- syntactic language models
- caching models
- recently used words are more likely to appear

$$
P_{\text {CACHE }}(w \mid \text { history })=\lambda P\left(w_{i} \mid w_{i-2} w_{i-1}\right)+(1-\lambda) \frac{c(w \in \text { history })}{\mid \text { history } \mid}
$$

- these perform very poorly for speech recognition (why?)

