TTIC 31190: Natural Language Processing – Prerequisite Quiz

Problem 1 (40 pts) A bag contains 3 red balls and 2 blue balls. All balls have the same probability of being drawn. Please beiefly explain your answer for each case.

1. (10 pts) We draw a ball from the bag. What is the probability that the ball is red?

   $P(red) = \frac{\#\ red\ balls}{\#\ red\ balls + \#\ blue\ balls} = \frac{3}{3+2} = \frac{3}{5}$

2. (15 pts) We draw a ball from the bag. Without putting back the first ball, we draw another ball from the bag. What is the probability that the second ball is red, if the first ball is red?

   $P(second = red \mid first = red) = \frac{\#\ red\ balls - 1}{\#\ red\ balls + \#\ blue\ balls - 1} = \frac{2}{3+2-1} = \frac{1}{2}$

3. (15 pts) We draw a ball from the bag. Without putting back the first ball, we draw another ball from the bag. What is the probability that the second ball is red, no matter what the color of the first ball is?

$$\begin{aligned} P(second = red) =& P(second = red \mid first = red) \cdot P(first = red) \\ &+ P(second = red \mid first = blue) \cdot P(first = blue) \\ =& \frac{1}{2} \cdot \frac{3}{5} + \frac{3}{4} \cdot \frac{2}{5} = \frac{3}{10} + \frac{3}{10} = \frac{3}{5} \end{aligned}$$

Problem 2 (40 pts) We have a function $f(x), x \in [-10, 10]$. For the following cases, what value of x minimizes $f(x)$? What is the minimum value of $f(x)$? Please briefly explain your answer.

1. (15 pts) $f(x) = x$

   $f(x)$ is monotonically increasing when $x \in [-10, 10]$, so the minimum value is $f(-10) = -10$.

2. (15 pts) $f(x) = x^2 - 2x + 1$

   Let $\frac{df(x)}{dx} = 0$, we have $2x - 2 = 0 \Rightarrow x = 1$, so the minimum value is $f(1) = 0$.

3. (10 pts) $f(x) = -\log(|x - 1|)$

   Since $g(x) = \log(x)$ is a monotonically increasing function, we have

$$f(x) = \begin{cases} -log(1-x), & x < 1 \quad \textit{(monotonically increasing)} \\ -log(x-1), & x > 1 \quad \textit{(monotonically decreasing)} \end{cases}$$

   Comparing $f(-10)$ and $f(10)$, we have $f(-10) < f(10)$, so the minimum value is $f(-10) = -log11$.
   Note: f(x) is undefined when $x = 1$, so it's also correct if you answer the minimum doesn't exist.

Problem 3 (20 pts) What is the time complexity of the following algorithm, in terms of n? Please explain your answer.

```
def foo(a: list[int]):
    n = len(a)
    b = list()
    for i in range(n): # enumerate all elements in a
        while len(b) > 0 and b[-1] >= a[i]: # compare the last element in b with a[i]
            b.pop() # remove the last element with O(1) complexity
        b.append(a[i]) # append a[i] to the end of b with O(1) complexity
    return b
```

The time complexity is $\mathcal{O}(n)$: each element in a is pushed to b once, and each element in b is popped at most once.