TTIC 31250
An Introduction to the Theory of Machine Learning

Learning finite state environments

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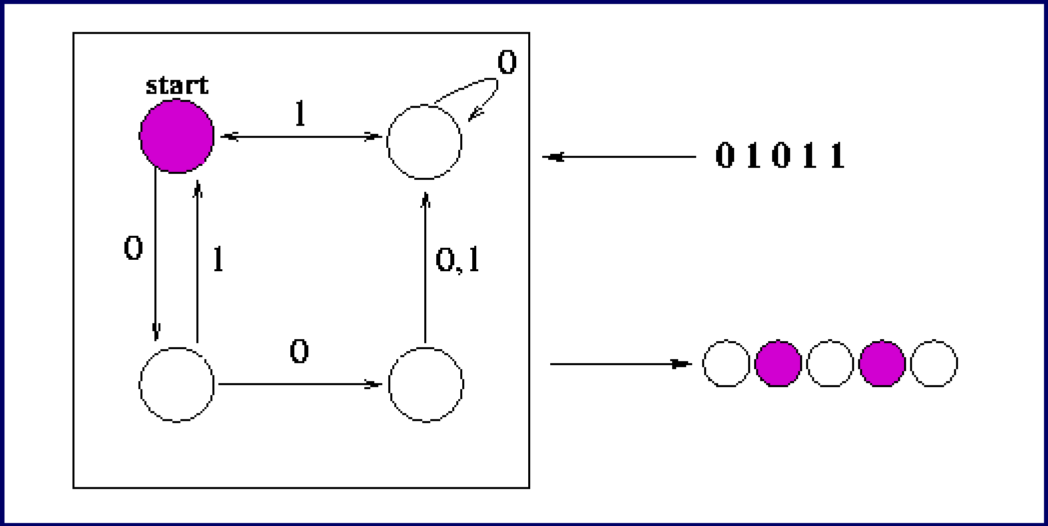
Consider the following setting

- Say we are a baby trying to figure out the effects our actions have on our environment...
  - Perform actions
  - Get observations
  - Try to make an internal model of what is happening.

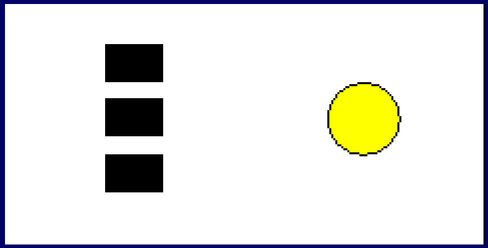
A model: learning a finite state environment

- Let's model the world as a deterministic finite automaton (DFA). We perform actions, we get observations.
- Our actions can also change the state of the world. # states is finite.



Another way to put it

- We have a box with buttons and lights.



- Can press the buttons, observe the lights.

  lights = f(current state)

  next state = g(button, current state)

- **Goal: learn predictive model of device.**

Relation to MDPs, POMDPs

MDP = Markov Decision Process
POMDP = Partially-observable MDP

- Compared to an MDP, this is harder in that multiple states may look identical but easier in that transitions are deterministic

- Like a POMDP with deterministic transitions.

- Goal is to learn the environment rather than gain reward.

Learning a DFA

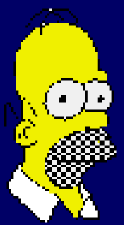
In the language of standard ML Theory models...

- Asking if we can learn a DFA from Membership Queries.
  - Issue of whether we have counterexamples (Equivalence Queries) or not.

    [for the moment, assume not]
  - Also issue of whether or not we have a reset button.

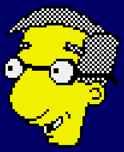
    [for now, assume yes]

Learning DFAs



This seems really hard. Can't tell for sure when world state has changed.

Let's look at an easier problem first: state = observation.



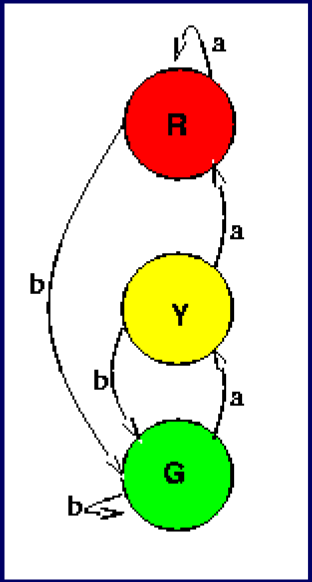
An example w/o hidden state

2 actions: a, b.

[disconnect projector]

An example w/o hidden state

2 actions: a, b.



Generic algorithm for lights=state:

- Build a model.
- While not done, find an unexplored edge and take it.

Now, let's try the harder problem!

Some examples

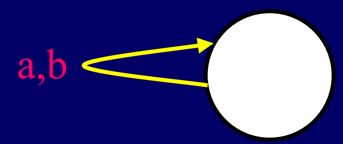
Example #1 (3 states)

Example #2 (3 states)

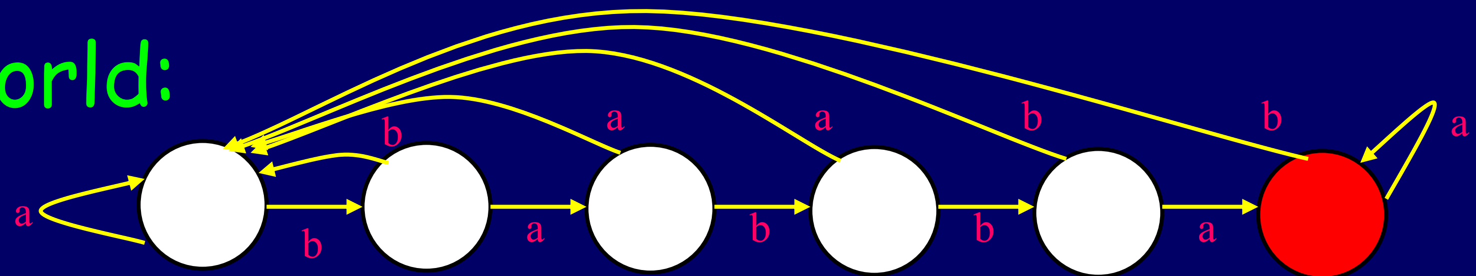
Can we design a procedure to do this in general?

One problem: what if we always see the same thing? How do we know there isn't something else out there?

Our model:

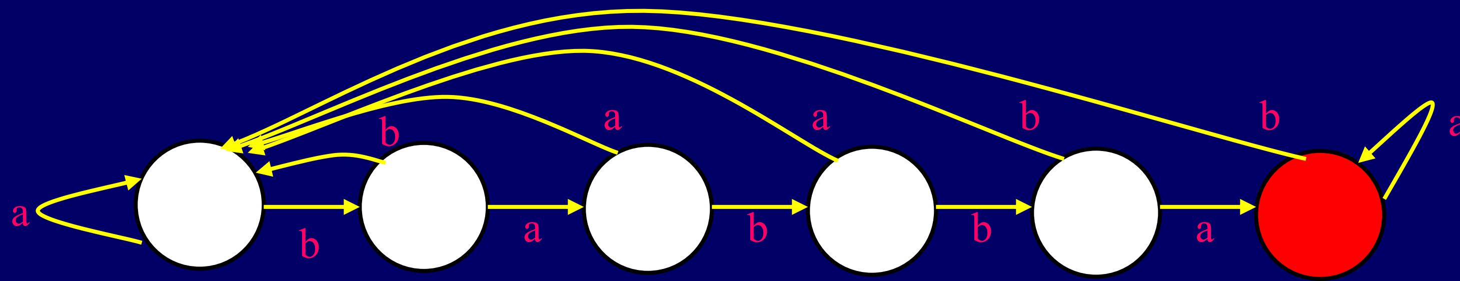


Real world:



Called "combination-lock automaton"

Can we design a procedure to do this in general?



Combination-lock automaton: basically simulating a conjunction.

This means we can't hope to efficiently come up with an exact model of the world from just our own experimentation. (I.e., MQs only).

How to get around this?

- Assume we can propose model and get counterexample. (MQ+EQ)
- Equivalently, goal is to be predictive. Any time we make a mistake, we think and perform experiments. (MQ+MB)
- Goal is not to have to do this too many times. For our algorithm, total # mistakes will be at most # states.

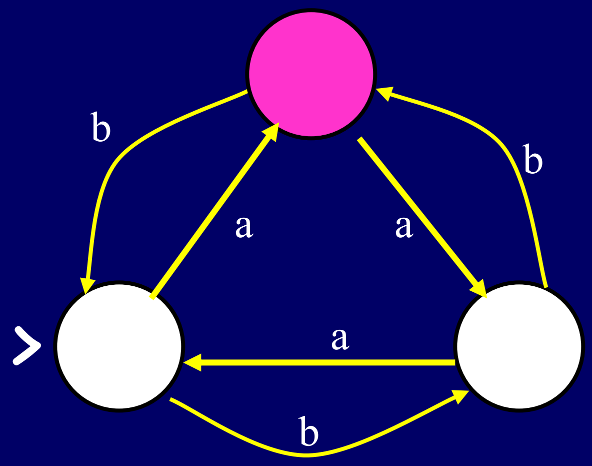
Algorithm by Dana Angluin

(with extensions by Rivest & Schapire)

- To simplify things, let's assume we have a RESET button. [Back to basic DFA problem]
- Can get rid of that using something called a "homing sequence" that you can also learn.

The problem (recap)

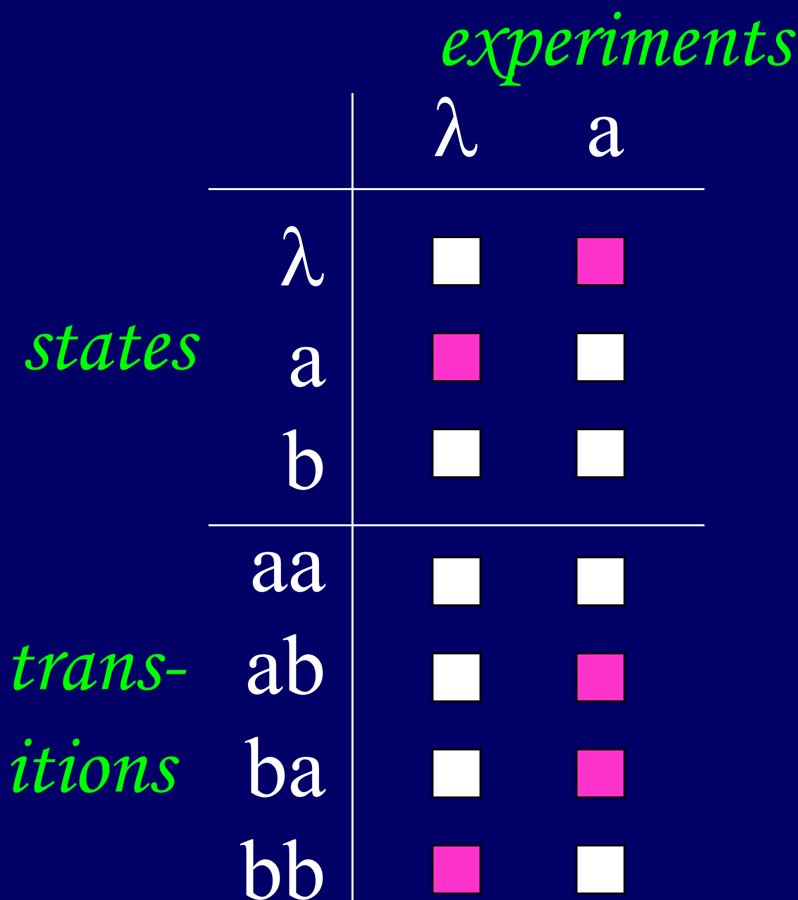
- We have a DFA:

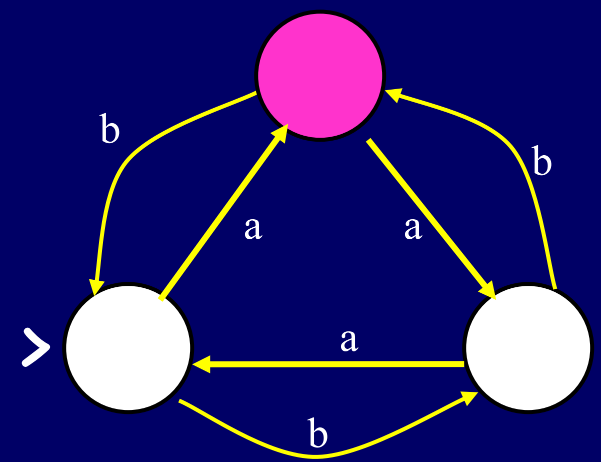


  - *observation = f(current state)*
  - *next state = g(button, prev state)*
- Can feed in sequence of actions, get observations. Then resets to start.
- Can also propose/field-test model. Get counterexample.

Key Idea

Key idea is to represent the DFA using a state/experiment table.





Every state has a name and a profile.

Key Idea

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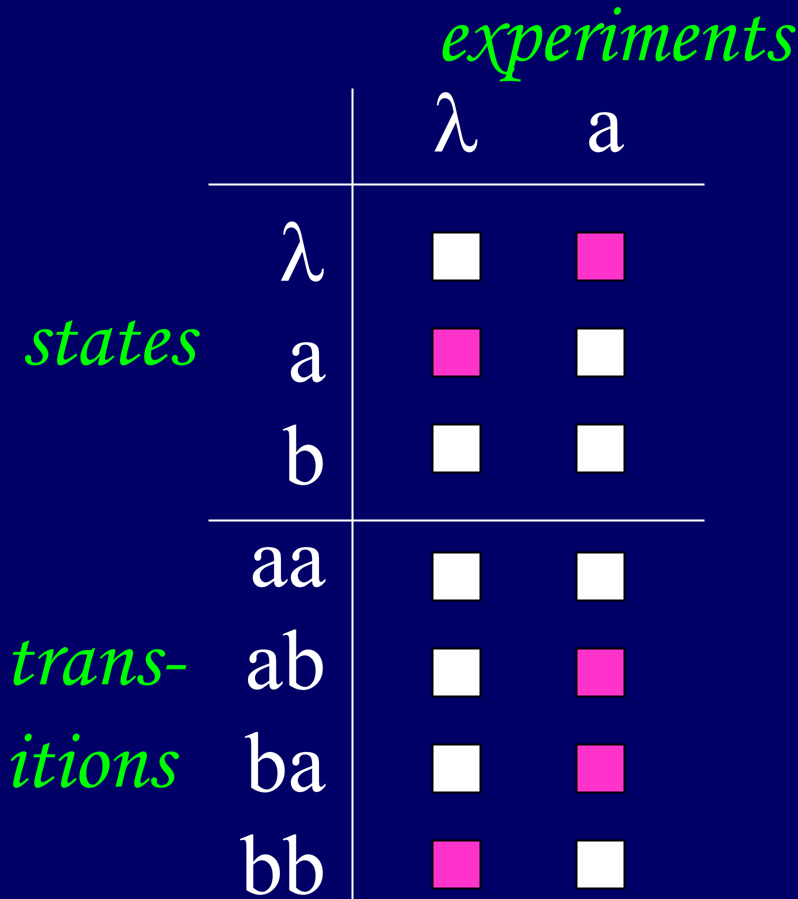
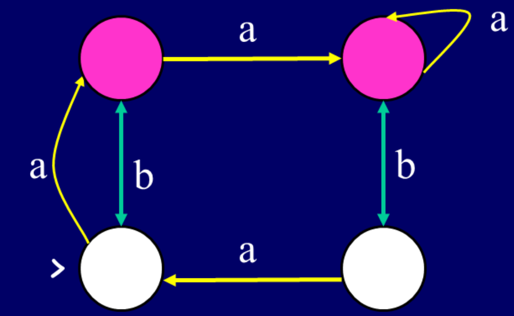


Guarantee will be: either this is correct, or else the world has > n states. In that case, need way of using counterexs to add new state to model.

The algorithm

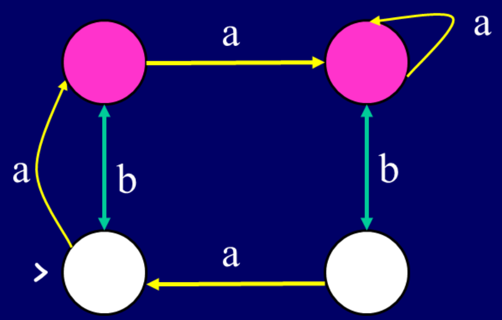
We'll do it by example...



(consider counterexample aaba)

The algorithm

We'll do it by example...



(consider counterexample aaba)

Algorithm (formally)

Begin with $S = \{\lambda\}, E = \{\lambda\}$.

1. Fill in transitions to make a hypothesis FSM.
2. While exists $s \in SA$ such that no $s' \in S$ has $row(s') = row(s)$, add s into S, and go to 1.
3. Query for counterexample z.
4. Consider all splits of z into (p_i, s_i), and replace p_i with its predicted equivalent $\alpha_i \in S$.
5. Find $\alpha_i r_i$ and $\alpha_{i+1} r_{i+1}$ that produce different observations.
6. Add r_{i+1} as a new experiment into E. go to 1.

Algorithm guarantees

If k actions, world has n states, then:

- At most n equivalence/mistake queries
- Final table has size $O(kn^2)$.
- So $O(kn^2)$ membership queries to fill in.
- Also $O(\log s)$ MQs per mistake where s is size of counterexample returned.