TTIC 31260 - Algorithmic Game Theory (Winter 2026)

Homework # 4 **Due: February 25, 2026**

Groundrules:

- You should work by yourself on the *exercises* but may work with a partner on the *problems* if you want. (Working together doesn't mean "splitting up the problems" though.) If you work with a partner, then write down who you are working with.
- If you've seen a problem before (sometimes I'll give problems that are "famous"), then say that in your solution. It won't affect your score, I just want to know. Also, if you use any sources other than the AGT book, write that down too. It's fine to look up a complicated sum or inequality, but please don't look up an entire solution.

Exercises:

1. **Bilateral Trade.** Use the characterization of IC direct-revelation mechansisms to show that there is no IC direct revelation mechanism for the bilateral trade problem with property that:
    1. A trade happens if $v_b > v_s$ and not if $v_b < v_s$.
    2. The trade occurs at a price in $[v_s, v_b]$ with no subsidy by the mechanism.
2. **Public Projects.** Use the characterization of IC direct-revelation mechansisms to show that there is no IC direct revelation mechanism for the problem of the government deciding whether to embark on a public project of cost C in a world with two citizens, with the property that: such that:
    1. If the two citizens' values on the project v_1 and v_2 satisfy $v_1 + v_2 > C$ then the project is undertaken and if $v_1 + v_2 < C$ then it's not.
    2. If the project is undertaken, then the citizens are in total charged at least C (i.e., no subsidy by the government), and citizen i is never charged more than v_i.
3. **Revenue and reserve prices.** Consider an auction with one item and two bidders. Recall that a 2nd-price auction sells the item to the highest bidder at a price equal to the second-highest bid. A 2nd-price auction with reserve price p does the following: if the highest bid is $\geq p$ then it sells the item to the highest bidder at the maximum of the second-highest bid and p; else it does not sell the item at all.

    Consider running a 2nd-price auction (with or without a reserve price) to two bidders whose valuations are known to be drawn independently at random from the uniform distribution on $[0, 1]$.

(a) What is the expected revenue of a second-price auction without a reserve price?

(b) What is the expected revenue of a second-price auction with reserve price $p = 1/2$?

You may use the fact that for independent random variables X and Y drawn uniformly from $[a, b]$, we have $E[\min(X, Y)] = a + (b - a)/3$.

Problems:

4. **Bidding languages.** Consider combinatorial auctions with m items. A valuation v is XOS if there exists a set of non-negative weight vectors $w_1, w_2, \ldots, w_t \in \mathbb{R}^m_{\geq 0}$ such that $v(S) = \max_j w_j \cdot x_S$ where $x_S \in \{0, 1\}^m$ is the indicator vector for set S. Show that $\{$submodular valuations$\} \subseteq \{$XOS valuations$\} \subseteq \{$subadditive valuations$\}$. In other words, any submodular valuation can be written as an XOS valuation, and any XOS valuation is subadditive. The first "$\subseteq$" relation is the hard one here.

   Hint: For thie first "$\subseteq$" relation, consider all possible permutations of the m items, and think about defining a weight vector for each of them.