TTIC 31260 - Algorithmic Game Theory (Winter 2026)

Homework # 3 **Due: February 16, 2026**

Groundrules:

- You should work by yourself on the *exercises* but may work with a partner on the *problems* if you want. (Working together doesn't mean "splitting up the problems" though.) If you work with a partner, then write down who you are working with.
- If you've seen a problem before (sometimes I'll give problems that are "famous"), then say that in your solution. It won't affect your score, I just want to know. Also, if you use any sources other than the AGT book, write that down too. It's fine to look up a complicated sum or inequality or whatever, but please don't look up an entire solution.

Exercises:

0. **Course Project.** One of the class requirements is to do a course project. This could be reading a recent paper related to class topics and explaining it in a 3-5 page writeup; it could be theoretically investigating a question related to class topics and writing up your thoughts in a 3-5 page report; or it could be conducting an experimental investigation and writing what you did and found in a 3-5 page report. Your project report is due on May 15. For this homework, your job is to think about it and write "I thought about it".

1. **Voting axioms.** Prove that any voting rule satisfying Condorcet Consistency must also satisfy Majority Consistency.

2. **Voting axioms II.** Arrow's impossibility theorem states that no social welfare function can satisfy all three of *unanimity*, *irrelevance of independent alternatives*, and *non-dictatorship*. Show that it is possible to achieve any two of the three.

3. **Incentive-compatibility.** Consider selling two identical printers by collecting bids and then giving one printer to the highest bidder at price equal to the second-highest bid, and giving the other printer to the second-highest bidder at price equal to the third-highest bid.

   Show that this mechanism is *not* incentive-compatible by giving an explicit set of valuations v_1, v_2, v_3 (assume three bidders) such that at least one bidder i would receive higher utility by misreporting some $v_i' \neq v_i$.

4. **VCG example.** Consider running the VCG mechanism with the Clarke pivot rule (i.e., the mechanism chooses the social-welfare-maximizing solution and each player is charged their externality) in the case of selling k identical printers. Specifically,

assume each agent i has value $v_i \geq 0$ on obtaining a printer and no additional value for receiving more than one printer. For concreteness, let's also assume all v_i are distinct.

Who gets the printers and what are they charged? Explain.

5. **VCG mechanism for a simple combinatorial auction.** Consider running a VCG mechanism with the Clarke pivot rule to auction two items, specifically a chair and a table. Assume we have three bidders who submit the following bids:

   - Bidder 1: \$10 for the chair, \$20 for the table, or \$25 for both together
   - Bidder 2: \$15 for the chair, \$15 for the table, or \$20 for both together
   - Bidder 3: \$10 for the chair, \$15 for the table, or \$30 for both together

   To whom will VCG allocate the items and how much will they be charged?