TTIC 31260 - Algorithmic Game Theory (Spring 2024)

Homework # 2 **Due: April 10, 2024**

Groundrules:

- You should work by yourself on the *exercises* but may work with a partner on the *problems* if you want. (Working together doesn't mean "splitting up the problems" though.) If you work with a partner, then write down who you are working with.
- If you've seen a problem before (sometimes I'll give problems that are "famous"), then say that in your solution. It won't affect your score, I just want to know. Also, if you use any sources other than the AGT book, write that down too. It's fine to look up a complicated sum or inequality or whatever, but please don't look up an entire solution.

Exercises:

1. **External regret vs Swap regret.**

   Consider playing T games of Rock-Paper-Scissors against an opponent who first plays Rock $T/3$ times, then plays Scissors $T/3$ times, then plays Paper $T/3$ times. Describe a sequence of (pure or mixed) strategies that would have zero external regret but $\Omega(T)$ swap-regret (and explain). Describe a second sequence of (pure or mixed) strategies with the same total payoff but that has no swap regret.

2. **Exercise 17.1 in the book.** The exact formula is not so enlightening, so instead of solving for the exact formula, argue what happens to the price of anarchy in the limit as $d \to \infty$.

3. **Exercise 17.2 in the book.**

Problems:

4. **Every Exact Potential Game is a Congestion Game.** In class, we showed that every congestion game is an exact potential game. Here you will show the converse (proven by Monderer and Shapley) that every exact potential game is also a congestion game.

   Specifically, assume you are given a game G with n players and exact potential function Φ. For convenience, you may assume that each player has the same number of action choices L. Your job is to define a congestion game such that in every state $s = (s_1, ..., s_n)$, the costs to each player are identical to their costs in that state in G. We will do this in two stages:

   (a) First, define a congestion game where in every state $s = (s_1, ..., s_n)$, each player incurs cost $\Phi(s)$. (See hints below)

(b) Now modify this game so that each player has the correct cost according to game G. (See hints below)

Hints: define 2^{nL} resources, where each resource is an $n \times L$ matrix of bits. Then define action j for player i as choosing all resources with a 1 in entry (i, j). What you now need to do is define cost functions for each resource, first to achieve goal (a) and then to achieve goal (b).

To aid in this, for each state s, let R_s^{min} be the minimal resource (the resource with the fewest 1's in it) such that all players have that resource in state s. What does R_s^{min} look like? Solve part (a) by only creating cost functions for these resources $\{R_s^{min}\}_{s \in S}$ (all other resources have cost of 0).

Now, to solve part (b), let $R_{s,i}^{max}$ be the maximal resource (the resource with the most 1's in it) such that only player i has that resource in state s. What does $R_{s,i}^{max}$ look like? Solve part (b) by taking your solution to part (a) and then also creating cost functions for these $R_{s,i}^{max}$ resources too. Note: these cost functions need not be monotone in the number of users. You should argue why your solution works and is well-defined.